Lemma A.3 (Uniform Kurdyka-Łojasiewicz Property) *Let Ω be a compact set and let $f : \mathbb{R}^n \to (-\infty, +\infty]$ be a proper and lower semicontinuous function. Assume that f is constant on Ω and satisfies the KŁ property at each point of Ω. Then there exists $\epsilon > 0$, $\eta > 0$, and $\varphi \in \Phi_\eta$, such that for all $\overline{\mathbf{u}}$ in Ω and all $\mathbf{u}$ in the following intersection*

$$\{\mathbf{u} \in \mathbb{R}^n : \mathrm{dist}(\mathbf{u}, \Omega) < \epsilon\} \cap \{\mathbf{u} \in \mathbb{R}^n : f(\overline{\mathbf{u}}) < f(\mathbf{u}) < f(\overline{\mathbf{u}}) + \eta\},$$

the following inequality holds:

$$\varphi'(f(\mathbf{u}) - f(\overline{\mathbf{u}}))\mathrm{dist}(\mathbf{0}, \partial f(\mathbf{u})) > 1.$$

Functions satisfying the KŁ property are general enough. Typical examples include: real polynomial functions, logistic loss function $\log(1+e^{-t})$, $\|\mathbf{x}\|_p$ $(p \geq 0)$, $\|\mathbf{x}\|_\infty$, and indicator functions of the positive semidefinite (PSD) cone, the Stiefel manifolds, and the set of constant rank matrices.

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