

References

- Anghileri, J. (1989). An investigation of young children's understanding of multiplication. *Educational Studies in Mathematics*, 20, 367–385.
- Bell, A., Fischbein, E., & Greer, B. (1984). Choice of operation in verbal arithmetic problems: The effects of number size, problem structure and context. *Educational Studies in Mathematics*, 15(2), 129–147.
- Bell, A., Greer, B., Grimison, L., & Mangan, C. (1989). Children's performance on multiplicative word problems: Elements of a descriptive theory. *Journal for Research in Mathematics Education*, 20(5), 434–449.
- Boaler, J. (2015). Fluency without fear: Research evidence on the best ways to learn math facts. Retrieved from youcubed.org.
- Boucher, D. (2014, September 16). Are 6 x 5 and 5 x 6 the same? *Math Coaches Corner*. Retrieved from <http://www.mathcoachescorner.com/2014/09/are-6-x-5-and-5-x-6-the-same/>.
- Bushart, B. (n.d.). Numberless word problems. *Teaching to the beat of a different drummer*. Retrieved from <https://bstockus.wordpress.com/numberless-word-problems/>.
- Carpenter, T. P. (1985). Learning to add and subtract: An exercise in problem solving. In *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 17–40). Hillsdale, NJ: Lawrence Erlbaum Associates.
- Carpenter, T. P., Ansell, E., Franke, M. L., Fennema, E., & Weisbeck, L. (1993). Models of problem solving: A study of kindergarten children's problem-solving processes. *Journal for Research in Mathematics Education*, 24, 428–441.
- Carpenter, T. P., Fennema, E., & Franke, M. L. (1996). Cognitively guided instruction: A knowledge base for reform in primary mathematics instruction. *The Elementary School Journal*, 97(1), 3–20.
- Carpenter, T. P., Fennema, E., Franke, M. L., Levi, L., & Empson, S. B. (2014). *Children's mathematics: Cognitively guided instruction* (2nd ed.). Portsmouth, NH: Heinemann.
- Carpenter, T. P., Hiebert, J., & Moser, J. M. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. *Journal for Research in Mathematics Education*, 12, 27–39.
- Confrey, J., & Smith, E. (1995). Splitting, covariation, and their role in the development of exponential functions. *Journal for Research in Mathematics Education*, 26, 66–86.
- De Corte, E., & Verschaffel, L. (1987). The effect of semantic structure on first graders' strategies for solving addition and subtraction word problems. *Journal for Research in Mathematics Education*, 18, 363–381.

- de Koning, B. B., Boonen, A. J. H., & van der Schoot, M. (2017). The consistency effect in word problem solving is effectively reduced through verbal instruction. *Contemporary Educational Psychology*, 49, 121–129.
- Devlin, K. (2008, June). It ain't no repeated addition. Mathematical Association of America. Retrieved from http://www.maa.org/external_archive/devlin/devlin_06_08.html.
- English, L. D. (1998). Children's problem posing within formal and informal contexts. *Journal for Research in Mathematics Education*, 29, 83–106.
- Fischbein, E., Deri, M., Nello, M. S., & Marino, M. S. (1985). The role of implicit models in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, 16, 3–17.
- Fosnot, C. T., & Dolk, M. (2001). *Young mathematicians at work: Constructing multiplication and division*. Portsmouth, NH: Heinemann.
- Franke, M. L. (2018, April). *How and why attention to student thinking supports teacher and student learning: The case of Cognitively Guided Instruction (CGI)*. Presented at the 2018 NCSM Annual Conference, Walter E. Washington Convention Center. Retrieved from <https://www.mathedleadership.org/events/conferences/DC2/index.html>.
- Garfunkel, S., & Montgomery, M. (Eds.). (2019). *GAIMME: Guidelines for assessment and instruction in mathematical modeling education* (2nd ed.). Philadelphia, PA: COMAP and SIAM. Retrieved from <https://siam.org/publications/reports/detail/guidelines-for-assessment-and-instruction-in-mathematical-modeling-education>.
- Gravemeijer, K. (1999). How emergent models may foster the constitution of formal mathematics. *Mathematical Thinking and Learning*, 1, 155–177.
- Gray, K. (2015, March 29). Commutativity in fraction multiplication. Retrieved from <https://kgmathminds.com/2015/03/29/commutativity-in-fraction-multiplication/>.
- Gutstein, E., & Romberg, T. A. (1995). Teaching children to add and subtract. *The Journal of Mathematical Behavior*, 14, 283–324.
- Heller, J. I., & Greeno, J. G. (1979). Information processing analyses of mathematical problem solving. In R. Lesh, M. Mierkiewicz, & M. Kantowski (Eds.), *Applied mathematical problem solving* (pp. 181–206). Columbus, OH: The Ohio State University. Retrieved from ERIC database (ED 180 816).
- Hutchins, P. (1986). *The doorbell rang*. New York: Greenwillow Books.
- Jenkins, S. (2011). *Actual size*. Boston, MA: HMH Books for Young Readers.
- Karp, K. S., Bush, S. B., & Dougherty, B. J. (2014). 13 rules that expire. *Teaching Children Mathematics*, 21, 18–25.
- Kelemanik, G., Lucenta, A., & Creighton, S. J. (2016). *Routines for reasoning: Fostering the mathematical practices in all students*. Portsmouth, NH: Heinemann.
- Kieren, T. E. (1976). On the mathematical, cognitive and instructional foundations of rational numbers. In R. A. Lesh & D. A. Bradhart (Eds.), *Number and measurement: Papers from a research workshop* (pp. 108–151). Retrieved from <https://files.eric.ed.gov/fulltext/ED120027.pdf>.
- Kouba, V. L., & Franklin, K. (1993). Multiplication and division: Sense making and meaning. In R. J. Jensen (Ed.), *Research Ideas for the Classroom: Early Childhood Mathematics* (pp. 103–126). Reston, VA: National Council of Teachers of Mathematics.
- Lamon, S. J. (2012). *Teaching fractions and ratios for understanding: Essential content knowledge and instructional strategies for teachers*. Mahwah, NJ: Routledge.
- Leinwand, S., Brahier, D. J., Huinker, D., Berry, R. Q., Dillon, F. L., Larson, M. R., ... Smith, M. S. (Eds.). (2014). *Principles to actions: Ensuring mathematical success for all*. Reston, VA: National Council of Teachers of Mathematics.
- Lesh, R. A., Post, T., & Behr, M. (1987). Representations and translations among representations in mathematics learning and problem solving. In C. Janvier (Ed.), *Problems of representations in the teaching*

- and learning of mathematics (pp. 33–40). Hillsdale, NJ: Lawrence Erlbaum Associates. Retrieved from http://www.cehd.umn.edu/ci/rationalnumberproject/87_5.html.
- McCallum, W., Daro, P., & Zimba, J. (n.d.). Progressions documents for the Common Core math standards. *The University of Arizona Institute for Mathematics and Education*. Retrieved from <http://ime.math.arizona.edu/progressions/>.
- Mulligan, J. T., & Mitchelmore, M. C. (1997). Young children's intuitive models of multiplication and Division. *Journal for Research in Mathematics Education*, 28, 309–330.
- Murphy, S. J. (1997). *Divide and ride*. New York: Harper Collins.
- Murphy, S. J. (2003). *The sundae scoop*. New York: Harper Collins.
- National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.
- National Governors Association Center for Best Practices and Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics*. Washington, DC: Author.
- Petitto, A. L., & Ginsburg, H. P. (1982). Mental arithmetic in Africa and America: Strategies, principles, and explanations. *International Journal of Psychology*, 17(1–4), 81–102.
- Philipp, R. A., & Hawthorne, C. (2015). Unpacking referent units in fraction operations. *Teaching Children Mathematics*, 22(4), 240–247. Retrieved from <https://doi.org/10.5951/teacchilmath.22.4.0240>.
- Piaget, J., & Inhelder, B. (1973). *The psychology of the child*. New York: Basic Books.
- Pólya, G. (1945). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press.
- Riley, M. S., Greeno, J. G., & Heller, J. I. (1984). Development of children's ability in arithmetic. In *Development of Children's Problem-Solving Ability in Arithmetic*. No. LRDC-1984/37. (pp. 153–196). Pittsburgh University, PA: Learning Research and Development Center, National Institute of Education.
- Rosenthal, A. K. (2011). *This plus that: Life's little equations*. New York: Harper Collins.
- Rudnitsky, A., Etheredge, S., Freeman, S. J. M., & Gilbert, T. (1995). Learning to solve addition and subtraction word problems through a structure-plus-writing approach. *Journal for Research in Mathematics Education*, 26, 467–486.
- SanGiovanni, J. J., & Milou, E. (2018). *Daily routines to jump-start math class, middle school: Engage students, improve number sense, and practice reasoning*. Thousand Oaks, CA: Corwin.
- Sayre, A. P., & Sayre, J. (2003). *One is a snail, ten is a crab*. London: Walker.
- Schliemann, A. D., Araujo, C., Cassundé, M. A., Macedo, S., & Nicéas, L. (1998). Use of multiplicative commutativity by school children and street sellers. *Journal for Research in Mathematics Education*, 29, 422–435.
- Scieszka, J., & Smith, L. (1995). *Math curse*. New York: Viking.
- Sowder, L. (2002). Story problems & students' strategies. In D. Chambers (Ed.), *Putting research into practice in the elementary grades* (pp. 21–23). Reston, VA: National Council of Teachers of Mathematics.
- Tang, G. (n.d.). Word problems. Retrieved from <https://gregtangmath.com/wordproblems>.
- Thompson, P. W. (2010). Quantitative reasoning and mathematical modeling. In L. L. Hatfield, S. Chamberlain & S. Belbase (Eds.), *New perspectives and directions for collaborative research in mathematics education* (Vol. 1, pp. 33–57). Laramie, WY: University of Wyoming.
- Usiskin, Z. (2012, April). *Unpacking mathematical understanding in the Common Core State Standards*. Presented at the National Council of Supervisors of Mathematics (NCSM), Philadelphia, PA. Retrieved from <https://www.mathedleadership.org/events/conferences/PA/allsessions.html>.
- Watanabe, T. (2003). Teaching multiplication: An analysis of elementary school mathematics teachers' manuals from Japan and the United States. *The Elementary School Journal*, 104, 111–125.