

# Contents

<i>Preface</i>		page ix
<i>List of Mathematical Symbols</i>		xiii
<b>1 What Is Statistical Mechanics For?</b>		1
1.1 What Is Statistical Mechanics?		1
1.2 Statistical Mechanics: The Rosetta Stone of Physics?		3
1.3 A Short Disclaimer: Statistical Mechanics Theory or Theories?		4
<b>2 Probability Theory</b>		6
2.1 Basic Ideas of Probability Theory		6
2.2 Clarifying the Notion of Event Space		8
2.3 Conditional Probability		10
2.4 Probability Space of Multiple Random Experiments		12
2.5 Notion of Independence		14
2.6 Contention on the Notion of Independence		15
2.7 Bayes' Theorem and Probability Updating		17
2.8 Basic Ideas on Discrete Random Variables		19
2.9 An Attempt at Continuous Random Variables		20
2.10 The Need for a More Formal Approach		21
2.11 Some Examples of Probability "Laws" Followed by Random Variables		26
2.12 Expectation Values and Applications		30
2.13 Moments and Cumulants of a Probability Distribution		31
2.14 Moment and Cumulant Generating Functions		34
2.15 The Necessity of Having a Prior		36
2.16 Jaynes' Least Biased Distribution Principle		37
2.17 End of Chapter 2 Exercises		41
<b>Complementary Material</b>		45
2.A Derivation of Eq. (2.4)		45
2.B Obtaining the Probability Density from a Moment Sequence: A Working Example		46
2.C A Generalised Form for the Information Entropy		47
2.D Derivation of Eq. (2.43)		49

<b>3</b>	<b>Gibbs Statistical Mechanics</b>	51
3.1	Motivation and Lexicon of Hamiltonian Mechanics for 1 Particle with 1 Degree of Freedom	51
3.2	Basic Ideas of Equilibrium Statistical Mechanics	56
3.3	Equilibrium Gibbs Ensembles for 1 Particle in 1 Dimension	63
3.4	Gibbs' Ensembles (in 1D) from Jaynes' Least Biased Distribution Principle	68
3.5	Gibbs Ensembles for 1 Particle in $d$ -Dimensions	72
3.6	Gibbs Ensembles for 1 Particle with $n_{\text{dof}}$ Degrees of Freedom	76
3.7	The Real Thing: Gibbs Ensembles for Many Particles	80
3.8	Some Useful Properties of Gibbs' Canonical Ensemble	84
3.9	Gibbs' Notions of Generic and Specific States	89
3.10	Fluctuating Particle Number: Grand and Petit Canonical Ensembles	94
3.11	End of Chapter 3 Exercises	97
	<b>Complementary Material</b>	102
3.A	Conservation of Probability and Continuity Equation	102
3.B	Derivation of Eq. (3.42)	104
3.C	Solution to Liouville's Equation for a Uniformly Accelerated Particle	106
3.D	Statistical Properties of a Sub-system in the Microcanonical Ensemble	109
<b>4</b>	<b>Connecting Thermodynamics with Statistical Mechanics</b>	111
4.1	Thermodynamic Theory: Basic Lexicon and the Zeroth Law	111
4.2	Thermodynamic Theory: The First Law	114
4.3	Thermodynamic Theory: The Second Law	117
4.4	Thermodynamic Theory: The Third Law	122
4.5	Thermodynamic Theory: Simple Compressible Systems	124
4.6	Thermodynamic Theory: Massieu Potentials	129
4.7	The Paradigmatic Model of an Ideal Gas	135
4.8	Connecting the Thermodynamic Entropy with the Information Entropy	141
4.9	Obtaining Other Thermodynamic Correspondences	146
4.10	Equivalence between Gibbs' Ensembles	150
4.11	End of Chapter 4 Exercises	155
	<b>Complementary Material</b>	160
4.A	Generic versus Specific States in Relation to Thermodynamics	160
4.B	Absolute Entropy without the Third Law of Thermodynamics	162
4.C	Gibbs' Paradox	163

<b>5</b>	<b>Quantum Statistical Mechanics</b>	168
5.1	Motivation	168
5.2	Notion of State in Quantum Mechanics	170
5.3	Time Evolution of Quantum States	188
5.4	Quantum Canonical Ensemble	191
5.5	Quantum Microcanonical Ensemble	195
5.6	Not So Simple: 1 Particle in a Box and the Semi-Classical Limit	197
5.7	Von Neumann Entropy	202
5.8	Correspondence between Thermodynamics and Quantum Statistical Mechanics	205
5.9	End of Chapter 5 Exercises	208
	<b>Complementary Material</b>	211
5.A	Quantum Mechanics of a Particle in a 1D Box	211
5.B	Estimation of the Error in the Semi-classical Approximation for a Particle in a Box	213
5.C	Classical Equilibrium Statistical Analog of a Quantum Equilibrium Statistical Model	215
<b>6</b>	<b>Quantum Statistical Mechanics for Systems of Identical Particles</b>	221
6.1	A Short Discussion on Terminology	221
6.2	Occupation Number Representation	222
6.3	Potential Restrictions on the Occupation Number from Allowed Irreducible Representations of the Permutation Group	226
6.4	Applying the Occupation Number Representation to Statistical Mechanics	234
6.5	Quantum Statistical Mechanics for Fermions	237
6.6	Quantum Statistical Mechanics for Bosons	240
6.7	Semi-classical Limit and Particle Degeneracy	242
6.8	End of Chapter 6 Exercises	246
	<b>Complementary Material</b>	249
6.A	Derivation of Propositions 6.8 and 6.10	249
6.B	Justification That Photons Have a Chemical Potential Equal to Zero	251
<b>7</b>	<b>Introduction to the Thermodynamical and Statistical Descriptions of Phase Transitions</b>	261
7.1	Preliminary Discussion	261
7.2	States of Matter and Phases of Matter	262
7.3	Thermodynamic Analysis of Isotherms of Real Gases	263
7.4	Statistical Mechanics of Magnetic Systems	274
7.5	Phase Transitions: A Quick Summary	283
7.6	End of Chapter 7 Exercises	284

<b>Complementary Material</b>	286
7.A Isobaric Ensemble and Bulk Modulus	286
<b>Appendix A Legendre Transformations</b>	288
<b>Appendix B Lagrangian and Hamiltonian Formalism for Fields</b>	293
<b>Appendix C Euler Gamma Function and Hyper Volume of a Sphere</b>	297
C.1 Basic Notions on Euler's Gamma Function	297
C.2 Hyper-Volume of a $n$ -Dimensional Ball	299
<b>Appendix D Vector Space, Dual Space and Hilbert Space</b>	301
D.1 $N$ -Dimensional Vector Space	301
D.2 Dual Vector Space, Metric Space and Hilbert Space	302
D.3 Bra-Ket Notation	303
<b>Appendix E Solutions to Warm-Up Exercises</b>	305
E.1 Chapter 2	305
E.2 Chapter 3	308
E.3 Chapter 4	315
E.4 Chapter 5	318
E.5 Chapter 6	322
E.6 Chapter 7	324
<i>References</i>	327
<i>Index</i>	336