

Cox and Giuli's Principles of Stellar Structure

Cox and Giuli's Principles of Stellar Structure has been the reference textbook for studies of the structure of stars for several decades. Its content and thorough discussion of all relevant aspects of stellar physics make it a much needed tool for scholars and students.

This new edition has been extended by four specialists in the field to take into account the most recent improvements relevant for the modeling and understanding of stars. New developments, such as the results from the Hipparcos mission, the solar model and the inclusion of particle diffusion in stellar models have been added to the original text.

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