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The Philosophy of Arithmetic, Husserl’s youthful work dedicated to a philosophical, or better, epistemological foundation of mathematics, shows the shift in his interests from more properly mathematical issues to those regarding the philosophy of mathematics. Husserl strives to understand and clarify what numbers and numerical relations are, a problem that he recasts in terms of the subjective origins of the foundational concepts of set theory and finite cardinal arithmetic. We will try to show that on the whole this work of Husserl’s does not deserve the criticism and reading neglect that it suffered from, ever since Frege published his well-known review.² Besides its bold contented psychologism, we find ideas and conceptualizations that not only were original then, but are still interesting today, such as those concerning the autonomy of the formal algorithmic aspects of abstract algebra and set-theoretic. Moreover, it is here that the Husserlian idea of a universal arithmetic receives its first formulation, the full elaboration of which will take at least ten more years, until his research on these topics reaches its mature form in 1901.

¹Husserl, *Philosophie der Arithmetik*. Mit ergänzenden Texten (1890–1901), Husserl XII, 1–263, hereafter cited as PAA. English translation cited as PAA.

²On Frege’s review “Husserl thinks that arithmetical knowledge is originally built up by founding on the most basic, everyday relations in a way that reflects our a priori cognitive development”

Frege 1894. Cp., for example, Quine 1949; Ficker 1952, 289; Bell 1956, 373. Among the commentators that give a positive re-evaluation of some aspects of the PAA: Furber 1952; Furber and Feys 1973 (especially Ch. VI; however, his focus is mainly on Husserl’s logical theories in his later work, in particular in the *Logical Investigations* and *Formal and Transcendental Logic*; Miles 1967; Willard 1974, 271, & 1974; Trosset 1996; Carr Hill 1994 & 2.

³See also *Leopoldine N. der Mathematik* (December/January 1901/02), PAA, App. 430–431, 704–705, 472, and the now famous edition Schwegler & Schwegler 2001. Willard 1984 argued against the Husserl’s view “the general perception of mathematical objects of the sort that a rigorous development of higher analysis is possible is a result of the fact that we have to abstract from the materiality of things alone.” However, he also notes that he writes that “these various