

Many numerical predictions of experimental phenomena in particle physics are made possible by exploiting the discovery that simplifications can happen when phenomena are investigated on short distance and time scales. This book provides a coherent exposition of the renormalization techniques underlying these calculations. After reminding the reader of some basic properties of field theories, examples are used to explain the problems to be treated. The technique of dimensional regularization and the renormalization group is then shown. Finally a number of key applications are demonstrated, culminating in the treatment of deeply inelastic scattering. Originally published in 1984, this title has been reissued as an Open Access publication on Cambridge Core.



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