

Providing a new perspective on quantum field theory, this book gives a pedagogical exposition of non-perturbative methods in relativistic quantum field theory and introduces the reader to modern research in theoretical physics. After describing non-perturbative methods in detail, it uses these methods to explore two-dimensional and four-dimensional gauge dynamics. The book concludes with a summary emphasizing the interplay between two- and four-dimensional gauge theories. Aimed at graduate students and researchers, this book covers topics from twodimensional conformal symmetry, affine Lie algebras, solitons, integrable models, bosonization, and 't Hooft model, to four-dimensional conformal invariance, integrability, large N expansion, Skyrme model, monopoles and instantons. Applications, first to simple field theories and gauge dynamics in two dimensions, and then to gauge theories in four dimensions and quantum chromodynamics in particular, are thoroughly described. Published originally in 2010, this title has been reissued as an Open Access publication on Cambridge Core.

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