

Instantons, or pseudoparticles, are solutions to the equations of motion in classical field theories on a Euclidean spacetime. Instantons are found everywhere in quantum theories as they have many applications in quantum tunnelling. Diverse physical phenomena may be described through quantum tunnelling, for example: the Josephson effect, the decay of meta-stable nuclear states, band formation in tight binding models of crystalline solids, the structure of the gauge theory vacuum, confinement in 2+1 dimensions, and the decay of superheated or supercooled phases. Drawing inspiration from Sidney Coleman's Erice lectures, this volume provides an accessible, detailed introduction to instanton methods, with many applications, making it a valuable resource for graduate students in many areas of physics, from condensed matter, particle and nuclear physics, to string theory. This title, first published in 2017, has been reissued as an Open Access publication on Cambridge Core.



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