

CAMBRIDGE MONOGRAPHS ON MATHEMATICAL PHYSICS

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From basic differential geometry through to the construction and study of black-hole and black-brane solutions in quantum gravity – via all the intermediate stages – this book provides a complete overview of the intersection of gravity, supergravity, and superstrings.

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