

This is a concise graduate level introduction to analytic functional methods in quantum field theory.

Functional integral methods provide relatively simple methods of solution to a wide range of problems in quantum field theory. After introducing the basic mathematical background, this book goes on to study applications and consequences of the formalism to the study of series expansions, measure, phase transitions, physics on spaces with non-trivial topologies, stochastic quantisation, fermions, QED, non-abelian gauge theories, symmetry breaking, the effective potential, finite temperature field theory, instantons and compositeness. Serious attention is paid to the shortcomings of the conventional formalism (e.g. problems of measure) as well as a detailed appraisal of the ambiguities of series summation.

This book will be of great value to graduate students in theoretical physics wishing to learn the use of functional integrals in quantum field theory. It will also be a useful reference for researchers in theoretical physics, especially those with an interest in experimental and theoretical particle physics and quantum field theory.

‘The subject is difficult but the arguments are presented in a clear and attractive manner . . . readers should derive much pleasure from the small remarks that shed so much light and make what could be a heavy-going account into a lively one.’

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