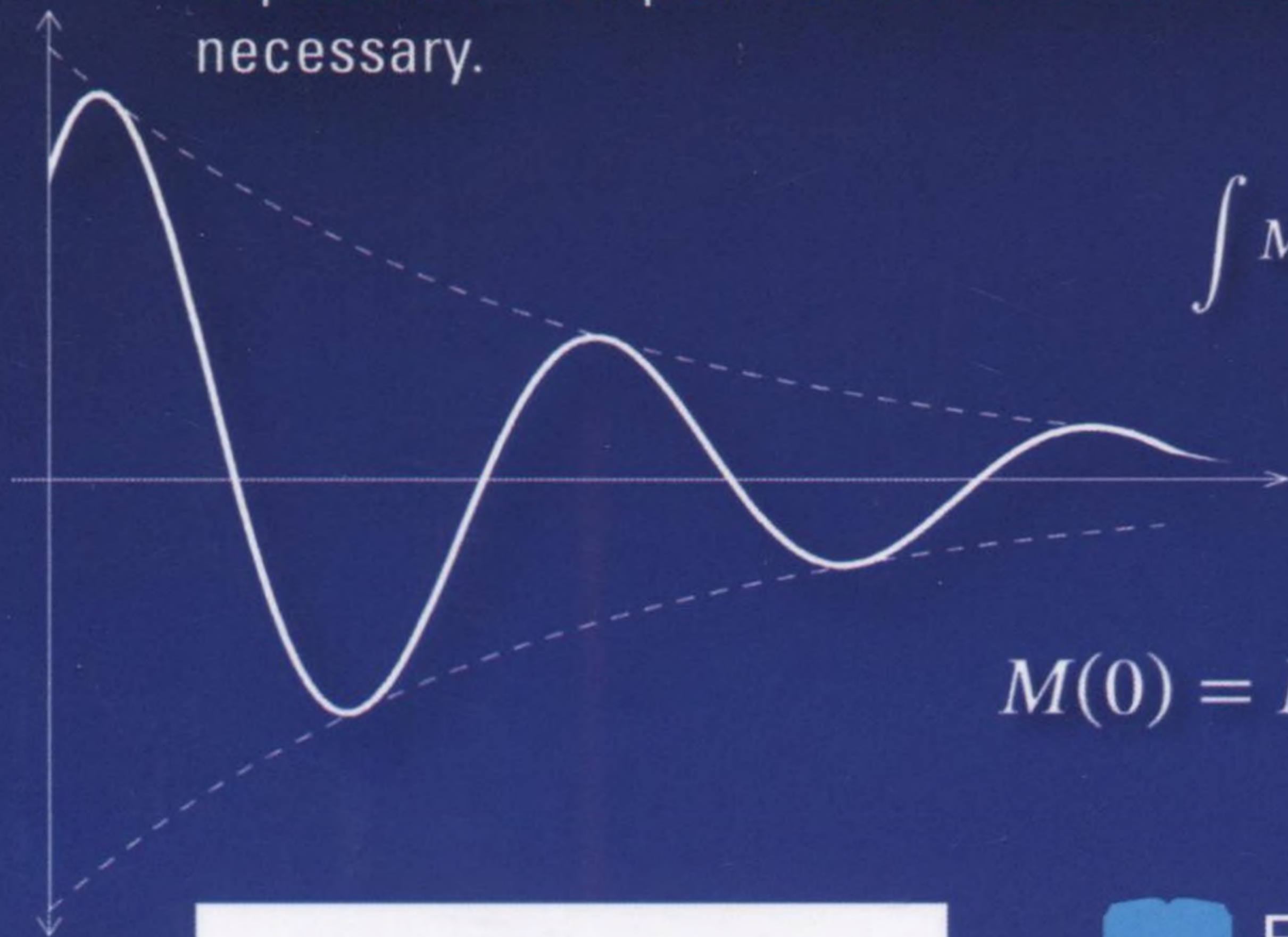


This book provides a collection of 130 fully solved problems, each derived from a mathematical model formulated in terms of an ordinary differential equation. As well as the problems, contextual descriptions are provided, grounding each model in its real-world setting. Applications to topics as diverse as physics, biology, engineering, and economics serve to underline the importance of differential equations in many areas of study.



The models are collected according to the underlying differential equation, making it a simple task to find the right model to implement within a class. The first chapter provides a catalog of first-order differential equations, which may be solved by elementary methods. The second chapter is concerned with models of exponential and logistic growth, including many models for population growth. Following this, there is a chapter on linear differential systems and equations. The book closes with a collection of problems arising from non-linear models. Here, qualitative methods such as phase portraits are emphasized.

The problems are designed for use in an undergraduate differential equations class. Knowledge of basic linear algebra and multivariable calculus is assumed, while an exposure to complex variables for some of the problems would be helpful, but not necessary.



$$\int M^{-2/3} dM = \alpha t + C \Rightarrow M(t) = \left(\frac{\alpha t}{3} + C\right)^3.$$

$$M(0) = M_0 = C^3 \Rightarrow M(t) = \left(\frac{\alpha t}{3} + \sqrt[3]{M_0}\right)^3.$$

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