

Written by the founder of *SIMIODE* (Systemic Initiative for Modeling Investigations and Opportunities with Differential Equations), this book provides a collection of resources that facilitate a modeling-first approach to teaching differential equations. Each of the forty-three chapters presents a unique, realistic, and classroom-tested modeling scenario, covering diverse topics such as chemical kinetics, stochastic processes, circuit tuning, and stadium design. The modules are standalone, meaning they can easily be introduced into an instructor's differential equations course. Through these activities, students discover how differential equations arise naturally and appreciate their importance.

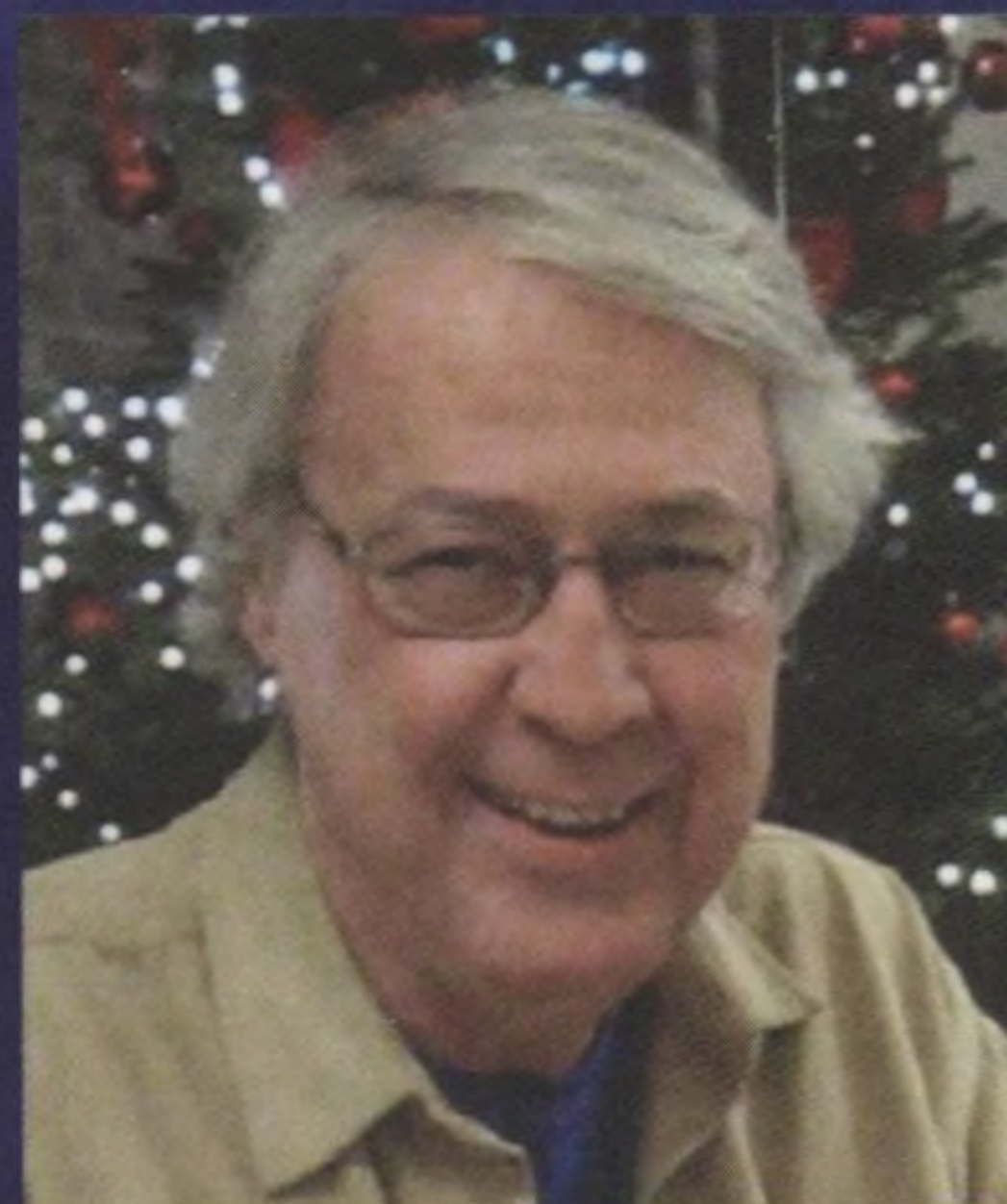


Photo by Brian J. Winkel

The material is presented in an informal style, taking the form of conversations between two colleagues discussing the underlying modeling scenarios. These colleagues share their methods, pedagogical approaches, and experiences, answering questions which arise about how to teach modeling. These dialogs also provide scaffolding, advice and reflections for faculty considering introducing active-learning opportunities into their classes.

Teaching Differential Equations with Modeling-First Scenarios offers a friendly introduction to modeling activities in the undergraduate classroom, with only prior exposure to derivatives being required. For the instructor, a desire to engender an attitude of experimentation, investigation, and excitement in their students is the most important prerequisite.

Materials supporting the scenarios will be available on the AMS Bookstore at www.ams.org/bookpages/clrm-78 and on the SIMIODE webpage.

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