

# Contents

## Preface

|          |                                          |    |
|----------|------------------------------------------|----|
| <b>1</b> | <b>Introduction</b>                      | 1  |
| 1.1      | General Setup                            | 1  |
| 1.2      | The Voter Model                          | 2  |
| 1.3      | The Contact Process                      | 4  |
| 1.4      | Ising and Potts Models                   | 7  |
| 1.5      | Phase Transitions                        | 10 |
| 1.6      | Variations on the Voter Model            | 12 |
| 1.7      | The Exclusion Process                    | 15 |
| 1.8      | Branching and Coalescing Particles       | 16 |
| 1.9      | Periodic Behavior                        | 18 |
| <b>2</b> | <b>Continuous-Time Markov Chains</b>     | 22 |
| 2.1      | Finite State Space                       | 22 |
| 2.2      | The Embedded Markov Chain                | 27 |
| 2.3      | Generator Construction                   | 28 |
| 2.4      | Lyapunov Functions                       | 31 |
| 2.5      | Poisson Point Sets                       | 33 |
| 2.6      | Poisson Construction of Markov Processes | 36 |
| 2.7      | An Example: ASEP                         | 41 |
| 2.8      | Local Maps                               | 44 |
| 2.9      | Systems of Finitely Many Particles       | 48 |
| <b>3</b> | <b>The Mean-Field Limit</b>              | 52 |
| 3.1      | Processes on the Complete Graph          | 52 |
| 3.2      | The Mean-Field Limit of the Ising Model  | 52 |
| 3.3      | Analysis of the Mean-Field Model         | 55 |
| 3.4      | Functions of Markov Processes            | 57 |
| 3.5      | The Mean-Field Contact Process           | 61 |
| 3.6      | The Mean-Field Voter Model               | 63 |
| 3.7      | Exercises                                | 65 |
| <b>4</b> | <b>Construction and Ergodicity</b>       | 67 |
| 4.1      | Introduction                             | 67 |
| 4.2      | Feller Processes                         | 68 |
| 4.3      | Poisson Construction                     | 76 |

|          |                                        |     |
|----------|----------------------------------------|-----|
| 4.4      | Finite Approximation and Perturbations | 83  |
| 4.5      | Generator Construction                 | 87  |
| 4.6      | Ergodicity                             | 94  |
| 4.7      | Application to the Ising Model         | 97  |
| 4.8      | Further Results                        | 100 |
| <b>5</b> | <b>Monotonicity</b>                    | 104 |
| 5.1      | The Stochastic Order                   | 104 |
| 5.2      | Monotone Interacting Particle Systems  | 106 |
| 5.3      | Positive Correlations                  | 109 |
| 5.4      | The Upper and Lower Invariant Laws     | 110 |
| 5.5      | The Contact Process                    | 113 |
| 5.6      | Other Examples                         | 115 |
| 5.7      | Exercises                              | 116 |
| <b>6</b> | <b>Duality</b>                         | 119 |
| 6.1      | Basic Definitions                      | 119 |
| 6.2      | Additive Systems                       | 121 |
| 6.3      | Additive Duality                       | 124 |
| 6.4      | Cancellative Systems and Their Duals   | 130 |
| 6.5      | Lloyd–Sudbury Duality                  | 136 |
| 6.6      | The Contact-Voter Model                | 141 |
| 6.7      | Invariant Laws of the Voter Model      | 143 |
| 6.8      | Homogeneous Invariant Laws             | 146 |
| 6.9      | Equality of Critical Points            | 151 |
| <b>7</b> | <b>Oriented Percolation</b>            | 153 |
| 7.1      | Introduction                           | 153 |
| 7.2      | Oriented Percolation                   | 154 |
| 7.3      | Survival                               | 156 |
| 7.4      | $K$ -Dependence                        | 158 |
|          | <i>References</i>                      | 163 |
|          | <i>Index</i>                           | 167 |