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$$\Gamma_{\text{L}}^{\text{L}} = \Gamma_{\text{th}}^{\text{L}} = \frac{1}{2\pi\alpha'} \frac{\Gamma_{\text{L}}^{\text{L}}}{\Gamma_{\text{th}}^{\text{L}}} \quad (\text{as a parameter, NO SUM})$$