Contents

Preface xi

SECTION I Quantum Physics

Chapter 1 Basic Properties 3

1.1 Classical Physics and Its Limitations 3
1.1.1 Particles 3
1.1.2 Waves 5
1.1.3 Particles and Waves 6
1.1.4 Complex Numbers 10
1.2 Statistical Properties 14
1.2.1 Probability 16
1.2.2 Normalization of the Wave Function 23
1.3 Physical Properties 25
1.3.1 Position Measurements 25
1.3.2 Momentum 27
1.3.3 Operators 29
1.3.4 Semiclassical Dynamics 30
Exercises 33

Chapter 2 Examples 36

2.1 Separation of Variables 36
2.1.1 Stationary States 36
2.1.2 Energy 37
2.1.3 Time-Dependent States 38
2.2 Infinite Square Well 39
2.2.1 Energy 39
2.2.2 States 40
2.2.3 Mathematical Properties 41
2.2.4 Statistical Properties 43
2.3 Harmonic Oscillator 44
2.3.1 Analytic Solutions 46
2.3.2 Algebraic Method 51
2.3.3 Time-Dependent Examples 57
2.3.4 Properties of States and Expectation Values 58
2.4 Free Particle 61
2.4.1 Stationary States 61
2.4.2 Wave Packets 62
2.4.3 Gaussian Wave Packet 63
2.4.4 Moving Particle 66
2.5 Scattering 67
2.5.1 Delta-Function Potential 69
2.5.2 Finite Square Well 75

2.5.3 Scattering Matrix 85
Exercises 88

Chapter 3 Quantum Mechanics on a Hilbert Space 91

3.1 Linear Algebra 91
3.1.1 Vectors 92
3.1.2 Inner Products 94
3.1.3 Linear Transformations 96
3.1.4 Eigenvectors and Eigenvalues 104
3.1.5 Hermitian Transformations 109
3.2 Function Spaces 110
3.2.1 Functions as Vectors 110
3.2.2 Operators as Linear Transformations 112
3.2.3 Uncertainty Relations and Commutators 119
3.2.4 Symmetries 123
Exercises 131

Chapter 4 Angular Momentum and Spin 134

4.1 Angular Momentum 135
4.1.1 Symmetry Transformations 136
4.1.2 Angular-Momentum Operators 137
4.1.3 Adjointness Properties 138
4.1.4 Algebraic Properties 139
4.1.5 Eigenvalues 141
4.1.6 Eigenvectors 144
4.1.7 Additional Properties of Angular Momentum 147
4.2 Radial Wave Equation 148
4.2.1 Radial Wave Function 149
4.2.2 The Hydrogen Example 151
4.3 Spin 163
4.3.1 Matrix Representations 164
4.3.2 Spin 1/2 165
4.3.3 Electron in a Magnetic Field 169
4.3.4 Identical Particles 170
Exercises 176

Chapter 5 Quantum Information 179

5.1 Quantum Correlations 179
5.2 Density Operators 181
5.2.1 Properties 182
5.2.2 Entropy 183
5.2.3 Bloch Sphere 184
5.2.4 Expectation Values 187
5.2.5 Time Dependence 188
5.2.6 Partial Trace and Entanglement 192
5.3 Operator-Valued Measures 193
5.3.1 Spectral Decomposition 194
5.3.2 Positive-Operator Valued Measures 194
5.3.3 Continuous Spectrum 196
5.4 Measurements 197

5.4.1 Definitions 198
5.4.2 Stern–Gerlach Experiments 203
5.4.3 The Measurement Problem 205
5.4.4 Representations 206
5.5 Time in Quantum Mechanics 212
5.5.1 Time as a Positive-Operator Valued Measure 213
5.5.2 Time Uncertainty 214
5.5.3 Time and Constraints 215
Exercises 219

SECTION II Relativity

Chapter 6 Basics of Gravity and Relativity 223
6.1 Non-Relativistic Gravity 223
6.1.1 Gravitational and Inertial Mass 223
6.1.2 Local Description 224
6.1.3 Gravitational Collapse 224
6.2 Newtonian Physics and the Relativity Principle 225
6.2.1 Inertial Frames 226
6.2.2 Line Element of Three-Dimensional Euclidean Space 227

Chapter 7 Special Relativity 230
7.1 Galilean Transformations 230
7.1.1 Partial Derivatives 231
7.1.2 Lorentz Transformations 232
7.2 Space-Time Geometry 233
7.2.1 Space-Time Diagrams 233
7.2.2 Distances in Space-Time 234
7.2.3 Consequences 237
7.3 Relativistic Dynamics 255
7.3.1 Linear Algebra 255
7.3.2 Dynamics 261
7.3.3 Light 267
7.3.4 Observables in Local Inertial Frames 271
Exercises 275

Chapter 8 Curved Spaces 279
8.1 Curvature 279
8.2 Space-Time Geometry and Gravity 282
8.3 Curved Space-Time from Newton's Potential 285
8.4 Metric Tensor 287
8.4.1 Coordinate Transformations 287
8.4.2 Singularities 288
8.4.3 Equivalence Principle 289
8.4.4 Causal Properties 290
8.5 Cosmology 290
8.5.1 Space-Time Analysis 291
8.5.2 Cosmological Models 292
8.5.3 Isotropic Singularity Theorem 295

8.5.4 Quantum Matter in the Early Universe 298
8.6 Geometrical Properties 303
8.6.1 Coordinate Lines and Induced Metrics 303
8.6.2 Spheres 305
8.6.3 Vectors in Curved Space-Time 307
8.6.4 Abstract Index Notation 308
8.6.5 Geodesics 312
8.6.6 Symmetry 320
8.6.7 Singularities 323
Exercises 328

Chapter 9 General Relativity 332
9.1 Curvature Tensors 332
9.1.1 Cancellations 332
9.1.2 Covariant Derivative 335
9.1.3 Einstein's Equation 340
9.2 General Relativity in Hamiltonian Form 347
9.2.1 Metric Components 348
9.2.2 Constraints 351
9.2.3 Symmetry 354
9.2.4 Spherically Symmetric Vacuum Solutions 356
9.3 Gauss–Codazzi Relations 359
9.3.1 Extrinsic Curvature 359
9.3.2 Projections 360
9.3.3 Ricci Scalar of Spherically Symmetric Spaces 362
Exercises 363

Chapter 10 Vacuum Solutions 365
10.1 Geodesics in the Schwarzschild Space-Time 366
10.1.1 Relativistic Corrections to Newton's Potential 366
10.1.2 Effective Potential 367
10.1.3 Escape Velocity 370
10.1.4 Kepler's Third Law 372
10.1.5 Perihelion Shift 373
10.1.6 Light 375
10.2 Black Holes 378
10.2.1 Eddington–Finkelstein Coordinates 378
10.2.2 Causal Structure 379
10.2.3 Rotating Black Holes 386
Exercises 388

SECTION III Relativity and Quantum Physics

Chapter 11 Quantum Mechanics with Newton's Potential 393
11.1 Newtonian Phase Shifts 393
11.2 Operators for Newton's Potential 395

Chapter 12 Relativistic Quantum Mechanics 397

12.1 Relativistic Phase Space and Its Quantization 397
12.1.1 Constraints and Observables 397
12.1.2 Hilbert Spaces 400
12.1.3 Superselection and Antimatter 402
12.2 Quantum Reference Frames 403
12.2.1 Frames for Relativistic Particles 403
12.2.2 Quantum Cosmological Models 404

Chapter 13 Quantum Gravity 407

13.1 Dimensional Arguments and Effective Field Theory 407
13.2 Approaches 409
13.2.1 Perturbative Quantum Gravity 409
13.2.2 Asymptotic Safety 410
13.2.3 Causal Dynamical Triangulations 410
13.2.4 Causal Sets 410
13.2.5 Non-Commutative and Non-Associative Geometries 411
13.2.6 String Theory 411
13.2.7 Canonical Quantum Gravity 416
13.2.8 Loop Quantum Gravity 417
13.3 Outlook 421

Index **427**