"This book is a real achievement! It explains the proof methods, semantical theories, and the latest results in the field, and the author puts forward his own view of relevant logic. It's constructed in such a way that it can be taught at various levels, and it's an excellent reference work as well as a fantastic teaching tool."

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"The author begins by promising to get the reader up to speed on relevant logic—and delivers brilliantly with his frame-based approach. Both enjoyable and illuminating."

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Relevant logics are non-classical logics that require the antecedent and consequent of implications to be relevantly related. They are paraconsistent logics, that is, they are able to robustly handle contradictory information. The field of non-classical logics is rapidly expanding, particularly with the addition of modalities and quantifiers. This is the first book to systematically develop a basic frame theory of relevant logics that includes both modal and quantified extensions. It includes sections comparing features of relevant logics with other, more common logics used in philosophy; examples and exercises to make the material more accessible; and an extensive bibliography. It also includes philosophical discussion of many aspects of relevant logics and highlights several directions for future research, both philosophical and formal.

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