Major shifts in the field of model theory in the twentieth century have seen the development of new tools, methods, and motivations for mathematicians and philosophers. In this book, John T. Baldwin places the revolution in its historical context from the ancient Greeks to the last century, argues for local rather than global foundations for mathematics, and provides philosophical viewpoints on the importance of modern model theory for both understanding and undertaking mathematical practice. The volume also addresses the impact of model theory on contemporary algebraic geometry, number theory, combinatorics, and differential equations. This comprehensive and detailed book will interest logicians and mathematicians as well as those working on the history and philosophy of mathematics.

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