CSLI Lecture Note Number 232

EVERYTHING WE DO is driven by information, but the dynamic process of acquiring information and adjusting our views based on it has seldom been put at center place in logic, and there are few textbooks introducing readers to this line of work. The present book introduces the reader to some major logical techniques for studying information dynamics and interaction between agents, based on a variety of intuitive scenarios as well as recent research. Readers will learn styles of modeling, as well as the basic features of the new logical systems emanating from this line of work. All the separate strands in the book are brought together in a comprehensive study of information flow and action in games, forming a logic-based complement to standard perspectives in game theory. The book contains pointers throughout to connections of this logical research program with topics in philosophy, computer science and cognitive science.

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