This revised and considerably expanded edition of *An Introduction to Non-Classical Logic* brings together a wide range of topics, including modal, tense, conditional, intuitionist, many-valued, paraconsistent, relevant and fuzzy logics. Part I, on propositional logic, is the old Introduction, but contains much new material. Part II is entirely novel, and covers quantification and identity for all the logics in Part I. The material is unified by the underlying theme of world semantics. All of the topics are explained clearly and accessibly, using devices such as tableau proofs, and their relation to current philosophical issues and debates is discussed. Students with a basic understanding of classical logic will find this book an invaluable introduction to an area that has become of central importance in both logic and philosophy. It will also interest people working in mathematics and computer science who wish to know about the area.

'Graham Priest's *Introduction to Non-Classical Logic* made this fascinating material on alternative logics accessible to my students for the very first time. The very welcome new edition extends the range of what is addressed to include important questions about quantification for modal logic, and the other systems as well.'

Tony Roy, California State University, San Bernardino

Praise for the first edition:

'Clear, self-contained, generously complete: this is bound to be the classic on non-classical logics for many years to come.' **Achille Varzi,** Columbia University

'... for anyone who wants to explore the non-classical systems, it is the only book of its kind and could not be more highly recommended.'

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