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This book introduces an important group of logics that have come to be known under the umbrella term 'substructural'. Substructural logics have independently led to significant developments in philosophy, computing and linguistics. *An Introduction to Substructural Logics* is the first book to systematically survey the new results and the significant impact that this class of logics has had on a wide range of fields. The following topics are covered:

- Proof Theory
- Propositional Structures
- Frames
- Decidability

Both students and professors of philosophy, computing, linguistics and mathematics will find this book an important addition to their reading.

Greg Restall is Senior Lecturer in Philosophy at Macquarie University in Sydney, Australia.

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