
Contents

Preface to first edition	v
Preface to second edition	ix
List of Figures	xvii
1 Orthogonal Series	1
1.1 General theory	1
1.2 Examples	5
1.2.1 Trigonometric system	6
1.2.2 Haar system	10
1.2.3 The Shannon system	12
1.3 Problems	15
2 A Primer on Tempered Distributions	19
2.1 Intuitive introduction	20
2.2 Test functions	22
2.3 Tempered distributions	25
2.3.1 Simple properties based on duality	27
2.3.2 Further properties	29
2.4 Fourier transforms	30
2.5 Periodic distributions	32
2.6 Analytic representations	33
2.7 Sobolev spaces	35
2.8 Problems	35
3 An Introduction to Orthogonal Wavelet Theory	37
3.1 Multiresolution analysis	38

3.2	Mother wavelet	44
3.3	Reproducing kernels and a moment condition	53
3.4	Regularity of wavelets as a moment condition	55
3.4.1	More on example 3	59
3.5	Mallat's decomposition and reconstruction algorithm	64
3.6	Filters	65
3.7	Problems	70
4	Convergence and Summability of Fourier Series	73
4.1	Pointwise convergence	73
4.2	Summability	79
4.3	Gibbs phenomenon	81
4.4	Periodic distributions	84
4.5	Problems	87
5	Wavelets and Tempered Distributions	91
5.1	Multiresolution analysis of tempered distributions	92
5.2	Wavelets based on distributions	95
5.2.1	Distribution solutions of dilation equations	95
5.2.2	A partial distributional multiresolution analysis	99
5.3	Distributions with point support	100
5.4	Approximation with impulse trains	104
5.5	Problems	107
6	Orthogonal Polynomials	109
6.1	General theory	109
6.2	Classical orthogonal polynomials	114
6.2.1	Legendre polynomials	115
6.2.2	Jacobi polynomials	119
6.2.3	Laguerre polynomials	120
6.2.4	Hermite polynomials	121
6.3	Problems	126
7	Other Orthogonal Systems	129
7.1	Self adjoint eigenvalue problems on finite intervals	130
7.2	Hilbert-Schmidt integral operators	132
7.3	An anomaly: the prolate spheroidal functions	134
7.4	A lucky accident?	135
7.5	Rademacher functions	140
7.6	Walsh function	142
7.7	Periodic wavelets	143

7.7.1	Periodizing wavelets	144
7.7.2	Periodic wavelets from scratch	146
7.8	Local sine or cosine basis	150
7.9	Biorthogonal wavelets	154
7.10	Problems	159
8	Pointwise Convergence of Wavelet Expansions	161
8.1	Reproducing kernel delta sequences	162
8.2	Positive and quasi-positive delta sequences	163
8.3	Local convergence of distribution expansions	169
8.4	Convergence almost everywhere	172
8.5	Rate of convergence of the delta sequence	173
8.6	Other partial sums of the wavelet expansion	177
8.7	Gibbs phenomenon	178
8.8	Positive scaling functions	181
8.8.1	A general construction	181
8.8.2	Back to wavelets	182
8.9	Problems	186
9	A Shannon Sampling Theorem in Wavelet Subspaces	187
9.1	A Riesz basis of V_m	188
9.2	The sampling sequence in V_m	189
9.3	Examples of sampling theorems	191
9.4	The sampling sequence in T_m	195
9.5	Shifted sampling	197
9.6	Gibbs phenomenon for sampling series	199
9.6.1	The Shannon case revisited	201
9.6.2	Back to wavelets	201
9.7	Irregular sampling in wavelet subspaces	212
9.8	Problems	214
10	Extensions of Wavelet Sampling Theorems	217
10.1	Oversampling with scaling functions	218
10.2	Hybrid sampling series	223
10.3	Positive hybrid sampling	225
10.4	The convergence of the positive hybrid series	228
10.5	Cardinal scaling functions	232
10.6	Interpolating multiwavelets	240
10.7	Orthogonal finite element multiwavelets	242
10.7.1	Sobolev type norm	244
10.7.2	The mother multiwavelets	245

10.8 Problems	252
11 Translation and Dilation Invariance in Orthogonal Systems	255
11.1 Trigonometric system	255
11.2 Orthogonal polynomials	256
11.3 An example where everything works	257
11.4 An example where nothing works	258
11.5 Weak translation invariance	259
11.6 Dilations and other operations	265
11.7 Problems	267
12 Analytic Representations Via Orthogonal Series	269
12.1 Trigonometric series	270
12.2 Hermite series	274
12.3 Legendre polynomial series	280
12.4 Analytic and harmonic wavelets	282
12.5 Analytic solutions to dilation equations	286
12.6 Analytic representation of distributions by wavelets	287
12.7 Wavelets analytic in the entire complex plane	291
12.8 Problems	293
13 Orthogonal Series in Statistics	295
13.1 Fourier series density estimators	296
13.2 Hermite series density estimators	299
13.3 The histogram as a wavelet estimator	301
13.4 Smooth wavelet estimators of density	305
13.5 Local convergence	309
13.6 Positive density estimators based on characteristic functions	310
13.7 Positive estimators based on positive wavelets	312
13.7.1 Numerical experiment	316
13.8 Density estimation with noisy data	318
13.9 Other estimation with wavelets	322
13.9.1 Spectral density estimation	322
13.9.2 Regression estimators	324
13.10 Threshold Methods	324
13.11 Problems	326

14 Orthogonal Systems and Stochastic Processes	329
14.1 K-L expansions	329
14.2 Stationary processes and wavelets	332
14.3 A series with uncorrelated coefficients	335
14.4 Wavelets based on band limited processes	341
14.5 Nonstationary processes	345
14.6 Problems	349
Bibliography	351
Index	363