

Contents

Preface	xiii
Introduction	xv
I. Spaces and Lattices of Open Sets	
1. Sober spaces	2
2. The axiom T_D : another case of spaces easy to reconstruct	5
3. Summing up	6
4. Aside: several technical properties of T_D -spaces	7
II. Frames and Locales. Spectra	
1. Frames	9
2. Locales and localic maps	11
3. Points	13
4. Spectra	15
5. The unit σ and spatiality	18
6. The unit λ and sobriety	20
III. Sublocales	
1. Extremal monomorphisms in Loc	23
2. Sublocales	25
3. The co-frame of sublocales	27
4. Images and preimages	29
5. Alternative representations of sublocales	30
6. Open and closed sublocales	33
7. Open and closed localic maps	36
8. Closure	39
9. Preimage as a homomorphism	41
10. Other special sublocales: one-point sublocales, and Boolean ones	42

11. Sublocales as quotients. Factorizing frames is surprisingly easy	44
IV. Structure of Localic Morphisms. The Categories $\mathbf{Loc}$ and $\mathbf{Frm}$	
1. Special morphisms. Factorizing in $\mathbf{Loc}$ and $\mathbf{Frm}$	49
2. The down-set functor and free constructions	51
3. Limits and a colimit in $\mathbf{Frm}$	54
4. Coproducts of frames	56
5. More on the structure of coproduct	59
6. Epimorphisms in $\mathbf{Frm}$	64
V. Separation Axioms	
1. Instead of T_1 : subfit and fit	73
2. Mimicking the Hausdorff axiom	79
3. I-Hausdorff frames and regular monomorphisms	83
4. Aside: Raney identity	86
5. Quite like the classical case: Regular, completely regular and normal	88
6. The categories $\mathbf{RegLoc}$, $\mathbf{CRegLoc}$, $\mathbf{HausLoc}$ and $\mathbf{FitLoc}$	92
VI. More on Sublocales	
1. Subspaces and sublocales of spaces	99
2. Spatial and induced sublocales	101
3. Complemented sublocales of spaces are spatial	103
4. The zero-dimensionality of $\mathcal{S}(L)^{op}$ and a few consequences	105
5. Difference and pseudodifference, residua	108
6. Isbell's Development Theorem	112
7. Locales with no non-spatial sublocales	116
8. Spaces with no non-induced sublocales	120
VII. Compactness and Local Compactness	
1. Basics, and a technical lemma	125
2. Compactness and separation	127
3. Kuratowski-Mrówka characterization	128
4. Compactification	131
5. Well below and rather below. Continuous completely regular frames	134
6. Continuous is the same as locally compact. Hofmann-Lawson duality	137
7. One more spatiality theorem	140

8.	Supercompactness. Algebraic, superalgebraic and supercontinuous frames	141
VIII. (Symmetric) Uniformity and Nearness		
1.	Background	145
2.	Uniformity and nearness in the point-free context	148
3.	Uniform homomorphisms. Modelling embeddings. Products	152
4.	Aside: admitting nearness in a weaker sense	157
5.	Compact uniform and nearness frames. Finite covers	159
6.	Completeness and completion	159
7.	Functoriality. CUniFrm is coreflective in UniFrm	164
8.	An easy completeness criterion	167
IX. Paracompactness		
1.	Full normality	169
2.	Paracompactness, and its various guises	173
3.	An elegant, specifically point-free, characterization of paracompactness	176
4.	A pleasant surprise: paracompact (co)reflection	180
X. More about Completion		
1.	A variant of the completion of uniform frames	183
2.	Two applications	188
3.	Cauchy points and the resulting space	189
4.	Cauchy spectrum	193
5.	Cauchy completion. The case of countably generated uniformities	199
6.	Generalized Cauchy points	201
XI. Metric Frames		
1.	Diameters and metric diameters	203
2.	Metric spectrum	207
3.	Uniform Metrization Theorem	211
4.	Metrization theorems for plain frames	214
5.	Categories of metric frames	218
XII. Entourages. Asymmetric Uniformity		
1.	Entourages	228
2.	Uniformities via entourages	231
3.	Entourages versus covers	233
4.	Asymmetric uniformity: the classical case	236

5.	Biframes	239
6.	Quasi-uniformity in the point-free context via paircovers	241
7.	The adjunction $\mathbf{QUnif} \rightleftarrows \mathbf{QUniFrm}$	248
8.	Quasi-uniformity in the point-free context via entourages	250
XIII. Connectedness		
1.	A few observations about sublocales	253
2.	Connected and disconnected locales	254
3.	Locally connected locales	258
4.	A weird example	260
5.	A few notes	267
XIV. The Frame of Reals and Real Functions		
1.	The frame $\mathfrak{L}(\mathbb{R})$ of reals	269
2.	Properties of $\mathfrak{L}(\mathbb{R})$	272
3.	$\mathfrak{L}(\mathbb{R})$ versus the usual space of reals	274
4.	The metric uniformity of $\mathfrak{L}(\mathbb{R})$	276
5.	Continuous real functions	278
6.	Cozero elements	284
7.	More general real functions	287
8.	Notes	294
XV. Localic Groups		
1.	Basics	297
2.	The category of localic groups	302
3.	Closed Subgroup Theorem	305
4.	The multiplication μ is open. The semigroup of open parts . .	306
5.	Uniformities	310
6.	Notes	312
Appendix I. Posets		
1.	Basics	315
2.	Zorn's Lemma	318
3.	Suprema and infima	319
4.	Semilattices, lattices and complete lattices. Completion	320
5.	Galois connections (adjunctions)	323
6.	(Semi)lattices as algebras. Distributive lattices	325
7.	Pseudocomplements and complements. Heyting and Boolean algebras	330

Appendix II. Categories

1. Categories	337
2. Functors and natural transformations	340
3. Some basic constructions	343
4. More special morphisms. Factorization	346
5. Limits and colimits	350
6. Adjunction	352
7. Adjointness and (co)limits	356
8. Reflective and coreflective subcategories	359
9. Monads	361
10. Algebras in a category	365
 Bibliography	 367
 List of Symbols	 381
 List of Categories	 385
 Index	 387

