

LECTURES ON PLANAR SOBOLEV EXTENSION DOMAINS

PEKKA KOSKELA

CONTENTS

1. Introduction	2
2. Reductions	5
3. Preliminaries from Real and Complex Analysis	8
4. Density	12
4.1. Decomposition of the core of Ω	12
4.2. Decomposition of the boundary layer of Ω	12
4.3. A partition of unity associated to the decomposition	13
4.4. Proofs of the density results	15
5. General necessary conditions for extendability	18
6. Linear extension operators	19
7. Extendability for $p \geq 2$	20
8. Proof of necessity when $1 < p < 2$	24
8.1. Necessity in the Jordan case	24
8.2. Inner extension	28
8.3. Proof of the general case	30
9. Proof of sufficiency when $1 < p < 2$	32
9.1. Transferring the condition to hyperbolic geodesics	32
9.2. Assigning Whitney squares for reflection	32
9.3. Constructing the extension operator	36
9.4. Proving the general case by exhausting with Jordan domains	42
References	43

Date: May 20, 2015.

2000 Mathematics Subject Classification. Primary 46E35.

Key words and phrases. Sobolev extension.

HOW TO FIND THE OPTIMAL PARTNER

LUBOŠ PICK

SETLIST

1. Getting better (Beatles 1967)	47
2. A nice pair (Pink Floyd 1973)	49
3. Space truckin' (Deep Purple 1973)	51
4. You rearrange me 'till I'm sane (Brain Damage/Eclipse) (Pink Floyd 1973)	57
5. Interstellar overdrive (Pink Floyd 1967)	61
6. Help me, operator (Memphis, Tennessee) (Chuck Berry 1959)	67
7. High hopes (Pink Floyd 1994)	68
8. Nothing compares 2 U (Prince 1984)	69
9. Gimme three steps (Lynyrd Skynyrd 1973)	73
10. Odds and sods (The Who 1974)	76
11. Highway to hell (AC/DC 1979)	79
12. Unbound (Avenged Sevenfold 2007)	82
13. Set the controls for the heart of the sun (Pink Floyd 1968)	85
14. Lipstick traces (UFO 1974)	98
References	106

1. GETTING BETTER (BEATLES 1967)

Sometimes, things improve. This happens in real life, and in mathematics, too. But then, an important question is, *how much* the things improve. Or, even more important, how far they *may* improve at all. The thing is, certain circumstances may cause restrictions on a possible improvement. Boundaries, obstacles, limitations. And, any questions involving a “how much” or a “how far” phrase can be stated (and answered) only if we have means to *measure* (hence assess) the improvement.

If somebody gives us a real number in $(0, 1)$, then we can always find a bigger one. Not only we can prove that there is one, but we can easily nail it down. If somebody gives us a real number in $(0, 1]$, then we might not necessarily be able to find a bigger one. On the other hand, we can now identify the *biggest* such number, which we couldn't have done in the former case. Since any two objects that can possibly come to the picture are comparable, we have tools for measuring improvement in this case. If it was the pure *size* of a number which mattered, and the bigger meant the better, then, instead of *the biggest* we could have said *the optimal*. Two other things could perhaps be noticed. When we accidentally hit the optimal (biggest) number

Date: May 20, 2015.

2000 Mathematics Subject Classification. 46E35.

Key words and phrases. Optimal range partner, optimal domain partner, sublinear operators, maximal operator, Laplace transform, Hardy integral operators, embeddings, rearrangement-invariant spaces, Lebesgue spaces, Lorentz spaces, Orlicz spaces, Sobolev spaces, reduction theorems, compact embeddings, fundamental functions, traces, Gaussian embeddings, isoperimetric function.

This research was supported by the grant P201-13-14743S of the Grant Agency of the Czech Republic.

LINEABILITY AND SPACEABILITY IN MATHEMATICS. LINEARITY IN NONLINEAR SETTINGS

JUAN B. SEOANE SEPÚLVEDA
Departamento de Análisis Matemático.
Universidad Complutense de Madrid (Spain).
E-mail: jseoane@ucm.es

Contents

1	Introduction	111
2	Real and Complex Analysis	115
3	Hypercyclicity and Chaos	137
4	Zeros of polynomials in Banach spaces	151
5	Some general results and remarks	156
	References	161

1 Introduction

Before starting, we would like to point out that the current survey notes are, simply, a partial expository document on the state of the art of the topic of lineability. We refer the interested reader to [83] or to the monograph [13] for a more complete and full detailed study of this topic.

Throughout history there have always been mathematical objects that have contradicted the intuition of the working mathematician. To cite some of these objects, let us recall the famous Weierstrass' Monster, Sierpiński's carpet, discontinuous additive functions (or Jones' functions), Peano curves, Cantor functions, or even the more modern differentiable nowhere monotone functions.

One may think that, once such an object is found, not many more like it can possibly exist. History has proven this last statement wrong. It is actually so wrong that, at the present time, the appearance of these exotic mathematical objects no longer comes as a surprise to mathematicians (for a quite complete