

*Fundamentals of Geophysical Fluid Dynamics* is a concise and accessible introduction to GFD for intermediate to advanced students of the physics, chemistry, and/or biology of Earth's fluid environment. This textbook was developed from the author's many years of teaching a first-year graduate course at the Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles. Readers are expected to be familiar with physics and mathematics at the level of general dynamics (mechanics) and partial differential equations.

#### Pre-publication praise

"... a delightfully refreshing introduction to graduate-level geophysical fluid dynamics. This well-written text includes a concise review of the needed applied mathematics, physics, and fluid dynamics. The text pulls examples not only from the atmospheres and oceans, but also from recent numerical studies and laboratory experiments in nonlinear dynamics, solitons, chaos and 2- and 3-dimensional turbulence, with an appropriate emphasis on their relevance to geophysical fluid dynamics. Some topics, for example geostrophic adjustment, are more clearly explained and are better physically motivated here than in any other text I have read. This book should not only be on the shelves of all geophysical fluid dynamicists, but also physicists, astronomers, and applied mathematicians."

**Professor Philip Marcus**, *Department of Mechanical Engineering,  
University of California, Berkeley*

"... a very good introductory text to geophysical fluid dynamics. Explanations of complex subjects are clear, concise, and insightful. Distracting and unnecessary details are avoided in discussions, and the organization of the material is well thought-out and logical ... ideal for use as a first exposure to the subject matter."

**Dr. Leif Thomas**, *School of Oceanography, University of Washington*

"Jim McWilliams' introductory book to the fundamentals of geophysical fluid dynamics is clearly written and well posed. The author relies on examples based on jets and vortices to introduce concepts such as turbulence, chaotic dynamics, bolus velocities, boundary layers, etc. that have not been extensively covered by existing textbooks. This book will therefore be very useful not only to graduate students, but also to scientists who are looking for a well-written reference book that is complementary to what is presently available."

**Dr. Eric P. Chassignet**, *Rosenstiel School of Marine and Atmospheric Science,  
Division of Meteorology and Physical Oceanography, University of Miami*

"McWilliams shows how the simplified models of geophysical fluid dynamics (GFD) can be used to explain the underlying physics in the complex turbulent flows in the Earth's atmosphere and oceans."

**Professor John A. Johnson**, *School of Mathematics, University of East Anglia*

Cover illustration: Landsat 7 satellite image of swirling clouds over Alexander Selkirk Island in the southern Pacific Ocean, as the result of a meteorological phenomenon known as a von Karman vortex. Rising precipitously from the surrounding waters, the island's highest point is nearly a mile (1.6 km) above sea level. As wind-driven clouds encounter this obstacle, they flow around it to form large, spinning eddies. Photo courtesy of NASA Goddard Space Flight Center.

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