

Theoretical and Mathematical Physics

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From Microphysics to Macrophysics

Methods and Applications of Statistical Physics

Volume I

Study Edition

This text not only provides a thorough introduction to statistical physics and thermodynamics but also exhibits the universality of the chain of ideas that leads from the laws of microphysics to the macroscopic behaviour of matter. A wide range of applications teaches students how to make use of the concepts, and many exercises will help to deepen their understanding. Drawing on both quantum mechanics and classical physics, the book follows modern research in statistical physics.

Volume I discusses in detail the probabilistic description of quantum or classical systems, the Boltzmann-Gibbs distributions, the conservation laws, and the interpretation of entropy as missing information. Thermodynamics and electromagnetism in matter are dealt with, as well as applications to gases, both dilute and condensed, and to phase transitions.

Volume II applies statistical methods to systems governed by quantum effects, in particular to solid state physics, explaining properties due to the crystal structure or to the lattice excitations or to the electrons. Liquid helium is discussed and radiative equilibrium and transport are studied. The last chapters are devoted to non-equilibrium processes and to kinetic equations, with many applications included.

This softcover edition accomodates the many requests to make this widely used and often cited classical text available again.

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