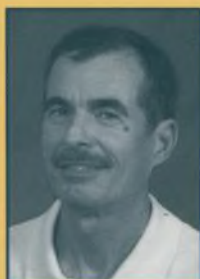


The mathematical theory of persistence answers questions such as which species, in a mathematical model of interacting species, will survive over the long term. It applies to infinite-dimensional as well as to finite-dimensional dynamical systems, and to discrete-time as well as to continuous-time semiflows.

This monograph provides a self-contained treatment of persistence theory that is accessible to graduate students. The key results for deterministic autonomous systems are proved in full detail such as the acyclicity theorem and the tripartition of a global compact attractor. Suitable conditions are given for persistence to imply strong persistence even for nonautonomous semiflows, and time-heterogeneous persistence results are developed using so-called “average Lyapunov functions”.

Applications play a large role in the monograph from the beginning. These include ODE models such as an SEIRS infectious disease in a meta-population and discrete-time nonlinear matrix models of demographic dynamics. Entire chapters are devoted to infinite-dimensional examples including an SI epidemic model with variable infectivity, microbial growth in a tubular bioreactor, and an age-structured model of cells growing in a chemostat.



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