

Contents

<i>How To Use This Book</i>	page xi
<i>Acknowledgments</i>	xiv

Part I Introduction 1

1	Concepts in Quantum Shannon Theory	3
1.1	Overview of the Quantum Theory	7
1.2	The Emergence of Quantum Shannon Theory	11
2	Classical Shannon Theory	26
2.1	Data Compression	26
2.2	Channel Capacity	35
2.3	Summary	49

Part II The Quantum Theory 51

3	The Noiseless Quantum Theory	53
3.1	Overview	54
3.2	Quantum Bits	55
3.3	Reversible Evolution	61
3.4	Measurement	68
3.5	Composite Quantum Systems	74
3.6	Summary and Extensions to Qudit States	89
3.7	History and Further Reading	96
4	The Noisy Quantum Theory	97
4.1	Noisy Quantum States	98
4.2	Measurement in the Noisy Quantum Theory	110
4.3	Composite Noisy Quantum Systems	112
4.4	Noisy Evolution	120
4.5	Summary	139
4.6	History and Further Reading	140
5	The Purified Quantum Theory	141
5.1	Purification	142
5.2	Isometric Evolution	143

5.3	Coherent Quantum Instrument	154
5.4	Coherent Measurement	155
5.5	History and Further Reading	156
Unit Quantum Protocols		157
Three Unit Quantum Protocols		159
6.1	Non-local Unit Resources	160
6.2	Protocols	162
6.3	Optimality of the Three Unit Protocols	171
6.4	Extensions for Quantum Shannon Theory	173
6.5	Three Unit Qudit Protocols	174
6.6	History and Further Reading	180
Coherent Protocols		181
7.1	Definition of Coherent Communication	182
7.2	Implementations of a Coherent Bit Channel	184
7.3	Coherent Dense Coding	185
7.4	Coherent Teleportation	187
7.5	The Coherent Communication Identity	189
7.6	History and Further Reading	190
The Unit Resource Capacity Region		191
8.1	The Unit Resource Achievable Region	191
8.2	The Direct Coding Theorem	195
8.3	The Converse Theorem	196
8.4	History and Further Reading	200
Tools of Quantum Shannon Theory		201
Distance Measures		203
9.1	Trace Distance	204
9.2	Fidelity	212
9.3	Relationships between Trace Distance and Fidelity	219
9.4	Gentle Measurement	223
9.5	Fidelity of a Noisy Quantum Channel	226
9.6	The Hilbert–Schmidt Distance Measure	230
9.7	History and Further Reading	231
Classical Information and Entropy		232
10.1	Entropy of a Random Variable	233
10.2	Conditional Entropy	237
10.3	Joint Entropy	239
10.4	Mutual Information	239
10.5	Relative Entropy	240

10.6	Conditional Mutual Information	241
10.7	Information Inequalities	243
10.8	Classical Information and Entropy of Quantum Systems	249
10.9	History and Further Reading	251
11	Quantum Information and Entropy	252
11.1	Quantum Entropy	253
11.2	Joint Quantum Entropy	258
11.3	Potential yet Unsatisfactory Definitions of Conditional Quantum Entropy	261
11.4	Conditional Quantum Entropy	263
11.5	Coherent Information	265
11.6	Quantum Mutual Information	267
11.7	Conditional Quantum Mutual Information	270
11.8	Quantum Relative Entropy	272
11.9	Quantum Information Inequalities	275
11.10	History and Further Reading	290
12	The Information of Quantum Channels	292
12.1	Mutual Information of a Classical Channel	293
12.2	Private Information of a Wiretap Channel	299
12.3	Holevo Information of a Quantum Channel	303
12.4	Mutual Information of a Quantum Channel	309
12.5	Coherent Information of a Quantum Channel	314
12.6	Private Information of a Quantum Channel	319
12.7	Summary	325
12.8	History and Further Reading	326
13	Classical Typicality	327
13.1	An Example of Typicality	328
13.2	Weak Typicality	329
13.3	Properties of the Typical Set	331
13.4	Application of Typical Sequences: Shannon Compression	333
13.5	Weak Joint Typicality	335
13.6	Weak Conditional Typicality	338
13.7	Strong Typicality	341
13.8	Strong Joint Typicality	350
13.9	Strong Conditional Typicality	352
13.10	Application: Shannon's Channel Capacity Theorem	358
13.11	Concluding Remarks	362
13.12	History and Further Reading	363
14	Quantum Typicality	364
14.1	The Typical Subspace	365
14.2	Conditional Quantum Typicality	375

14.3	The Method of Types for Quantum Systems	384
14.4	Concluding Remarks	387
14.5	History and Further Reading	387

The Packing Lemma

15.1	Introductory Example	388
15.2	The Setting of the Packing Lemma	389
15.3	Statement of the Packing Lemma	391
15.4	Proof of the Packing Lemma	393
15.5	Derandomization and Expurgation	398
15.6	History and Further Reading	400

The Covering Lemma

16.1	Introductory Example	401
16.2	Setting and Statement of the Covering Lemma	402
16.3	Proof of the Covering Lemma	404
16.4	History and Further Reading	406

Noiseless Quantum Shannon Theory

Schumacher Compression

17.1	The Information-Processing Task	415
17.2	The Quantum Data-Compression Theorem	417
17.3	Quantum Compression Example	418
17.4	Variations on the Schumacher Theme	420
17.5	Concluding Remarks	424
17.6	History and Further Reading	425

Entanglement Concentration

18.1	An Example of Entanglement Concentration	427
18.2	The Information-Processing Task	429
18.3	The Entanglement Concentration Theorem	430
18.4	Common Randomness Concentration	433
18.5	Schumacher Compression versus Entanglement Concentration	433
18.6	Concluding Remarks	440
18.7	History and Further Reading	441

Noisy Quantum Shannon Theory

Classical Communication

19.1	Naive Approach: Product Measurements at the Decoder	447
19.2	The Information-Processing Task	451
19.3	The Classical Capacity Theorem	453
19.4	Examples of Channels	456

19.5	Superadditivity of the Holevo Information	458
19.6	Concluding Remarks	460
19.7	History and Further Reading	463

20

Entanglement-Assisted Classical Communication

20.1	The Information-Processing Task	477
20.2	A Preliminary Example	479
20.3	The Entanglement-Assisted Classical Capacity Theorem	480
20.4	The Direct Coding Theorem	484
20.5	The Converse Theorem	484
20.6	Examples of Channels	493
20.7	Concluding Remarks	501
20.8	History and Further Reading	506

21

Coherent Communication with Noisy Resources

21.1	Entanglement-Assisted Quantum Communication	507
21.2	Quantum Communication	509
21.3	Noisy Super-Dense Coding	514
21.4	State Transfer	515
21.5	Trade-off Coding	518
21.6	Concluding Remarks	522
21.7	History and Further Reading	530

22

Private Classical Communication

22.1	The Information-Processing Task	531
22.2	The Private Classical Capacity Theorem	532
22.3	The Direct Coding Theorem	533
22.4	The Converse Theorem	536
22.5	Discussion of Private Classical Capacity	536
22.6	History and Further Reading	545

23

Quantum Communication

23.1	The Information-Processing Task	550
23.2	The No-Cloning Theorem and Quantum Communication	551
23.3	The Quantum Capacity Theorem	553
23.4	The Direct Coding Theorem	554
23.5	Converse Theorem	555
23.6	An Interlude with Quantum Stabilizer Codes	562
23.7	Example Channels	564
23.8	Discussion of Quantum Capacity	571
23.9	Entanglement Distillation	574
23.10	History and Further Reading	579

24	Trading Resources for Communication	585
24.1	The Information-Processing Task	586
24.2	The Quantum Dynamic Capacity Theorem	588
24.3	The Direct Coding Theorem	593
24.4	The Converse Theorem	596
24.5	Examples of Channels	606
24.6	History and Further Reading	616
25	Summary and Outlook	618
25.1	Unit Protocols	619
25.2	Noiseless Quantum Shannon Theory	619
25.3	Noisy Quantum Shannon Theory	620
25.4	Protocols Not Covered in This Book	623
25.5	Network Quantum Shannon Theory	624
25.6	Future Directions	625
Appendix A	Miscellaneous Mathematics	626
Appendix B	Monotonicity of Quantum Relative Entropy	633
	<i>References</i>	639
	<i>Index</i>	653