

The algebraic theory of automata was created by Schützenberger and Chomsky over 50 years ago and there has since been a great deal of development. Classical work on the theory of noncommutative power series has been augmented more recently to areas such as representation theory, combinatorial mathematics and theoretical computer science.

This book presents to an audience of graduate students and researchers a modern account of the subject and its applications. The algebraic approach allows the theory to be developed in a general form of wide applicability. For example, number-theoretic results can now be more fully explored, in addition to applications in automata theory, codes and noncommutative algebra. Much material, for example, Schützenberger's theorem on polynomially bounded rational series, and results on semisimple algebras, appear here for the first time in book form.

In sum, this is an excellent resource and reference for all those working in algebra, theoretical computer science and their areas of overlap.

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