

Computational Methods for GEODYNAMICS

Computational Methods for Geodynamics describes all the numerical methods typically used to solve problems related to the dynamics of the Earth and other terrestrial planets – including lithospheric deformation, mantle convection and the geodynamo.

It starts with a discussion of the fundamental principles of mathematical and numerical modelling, which is then followed by chapters on finite difference, finite volume, finite element and spectral methods; methods for solving large systems of linear algebraic equations and ordinary differential equations; data assimilation methods in geodynamics; and the basic concepts of parallel computing. The final chapter presents a detailed discussion of specific geodynamic applications in order to highlight key differences between methods and demonstrate their respective limitations. Readers learn when and how to use a particular method in order to produce the most accurate results.

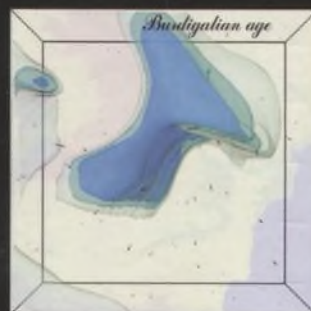
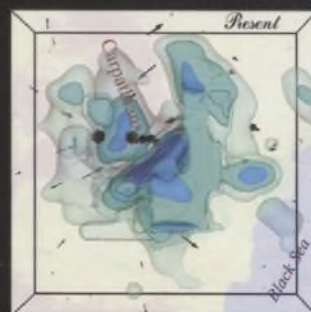
This combination of textbook and reference handbook brings together material previously only available in specialist journals and mathematical reference volumes, and presents it in an accessible manner, assuming only a basic familiarity with geodynamic theory and calculus. It is an essential text for advanced courses on numerical and computational modelling in geodynamics and geophysics, and an invaluable resource for researchers looking to master cutting-edge techniques. Links to online source codes for geodynamic modeling can be found at www.cambridge.org/zadeh.

"An outstanding synthesis of contemporary issues in geodynamics with a rigorous but highly accessible treatment of modern methods in numerical modeling. I have no doubt that this book will be an invaluable resource for students and researchers entering the field of computational geophysics for years to come."

PROFESSOR DAVID BERCOVICI, Yale University

"This is the most current and complete book on computational geodynamics. I would recommend this book to every aspiring student or researcher interested in computations."

PROFESSOR DAVID YUEN, University of Minnesota



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Geodynamics is the application of the basic principles of physics, chemistry and mathematics in understanding how the internal activity of the Earth results in all the geological phenomena and structures observed at the surface, including tectonic spreading and contraction, drift, mountain building, volcanoes, earthquakes, sedimentary basins, fluting, folding and more. Geodynamics also deals with how the Earth's internal activity and structure reveals itself externally in ways both geophysical, its gravitational and magnetic fields and geochemical, the mineralogy of its rocks and the tectonic convection of its rocks, atmosphere and oceans. The discipline of geodynamics did not even exist until about the early 1970s. The plate tectonic revolution was the impetus for the birth of the subject. Today, geodynamics goes beyond the Earth to include the activities and structures of other planets and moons in our solar system. While this aspect of the science could be termed planetary dynamics, it involves the same geodynamic processes that shape the Earth, though often with intriguingly different outcomes for the other bodies.

Mathematical modelling, which attempts to understand the processes quantitatively, lies at the heart of geodynamics. In the early years of the subject, analytic and semi-analytic methods were sufficient to gain insight into the workings of the Earth's interior. After four decades of progress in the subject it is, generally speaking, no longer possible to address the remaining questions with such simple models. Indeed, it has been necessary, for some time now, to employ sophisticated numerical computational models in order to further understanding of the complex dynamics of the Earth. Accordingly, researchers now entering the field of geodynamics need to acquire the skills to understand the numerical methods upon which contemporary geodynamic codes are based. An understanding of the methods is required not only for the intelligent use of existing codes but also to enable adaptations of the codes and future improvements in them. The present book responds to this need by thoroughly discussing the many different numerical schemes designed to provide approximate solutions to the ordinary and partial differential equations encountered in geodynamical problems and by explaining the fundamental principles behind the major code approaches. It is a book that goes far beyond the black box diffusion of numerical codes by providing the student with a deep understanding of the numerical approaches upon which the codes are based.

The book begins with an introductory chapter discussing the basic equations and the boundary and initial conditions of geodynamical problems. Subsequent chapters on the numerical approach to their solution. The subsequent chapters discuss in the whole code numerical schemes in detail, finite differences, finite volumes, finite elements, spectral methods, finite