

"*Taming the Unknown* is well written and informative, and will satisfy any reader with an interest in the history of algebra. Striking just the right balance between general overview and technical detail, this book is a pleasure to read."

—Joseph W. Dauben, City University of New York,
Graduate Center

"This original and high-quality book is a significant contribution to the history of mathematics. It will be useful to scholars doing research in the history of mathematics, as well as to a broader readership that includes mathematics teachers, advanced undergraduate or graduate students, and mathematicians."

—Leo Corry, Tel Aviv University

"This fine survey of the history of algebra is clearly and engagingly written. It will become the standard reference on this topic by virtue of its scholarship, coverage, and readability."

—Tom Archibald, Simon Fraser University

Contents

	Acknowledgments	xi
1	Prelude: What Is Algebra?	1
	Why This Book?	3
	Setting and Examining the Historical Parameters	4
	The Task at Hand	10
2	Egypt and Mesopotamia	12
	Proportions in Egypt	12
	Geometrical Algebra in Mesopotamia	17
3	The Ancient Greek World	33
	Geometrical Algebra in Euclid's <i>Elements</i> and <i>Data</i>	34
	Geometrical Algebra in Apollonius's <i>Conics</i>	48
	Archimedes and the Solution of a Cubic Equation	53
4	Later Alexandrian Developments	58
	Diophantine Preliminaries	60
	A Sampling from the <i>Arithmetica</i> : The First Three Greek Books	63
	A Sampling from the <i>Arithmetica</i> : The Arabic Books	68
	A Sampling from the <i>Arithmetica</i> : The Remaining Greek Books	73
	The Reception and Transmission of the <i>Arithmetica</i>	77
5	Algebraic Thought in Ancient and Medieval China	81
	Proportions and Linear Equations	82
	Polynomial Equations	90
	Indeterminate Analysis	98
	The Chinese Remainder Problem	100

6	Algebraic Thought in Medieval India	105
	Proportions and Linear Equations	107
	Quadratic Equations	109
	Indeterminate Equations	118
	Linear Congruences and the Pulverizer	119
	The Pell Equation	122
	Sums of Series	126
7	Algebraic Thought in Medieval Islam	132
	Quadratic Equations	137
	Indeterminate Equations	153
	The Algebra of Polynomials	158
	The Solution of Cubic Equations	165
8	Transmission, Transplantation, and Diffusion in the Latin West	174
	The Transplantation of Algebraic Thought in the Thirteenth Century	178
	The Diffusion of Algebraic Thought on the Italian Peninsula and Its Environs from the Thirteenth Through the Fifteenth Centuries	190
	The Diffusion of Algebraic Thought and the Development of Algebraic Notation outside of Italy	204
9	The Growth of Algebraic Thought in Sixteenth-Century Europe	214
	Solutions of General Cubics and Quartics	215
	Toward Algebra as a General Problem-Solving Technique	227
10	From Analytic Geometry to the Fundamental Theorem of Algebra	247
	Thomas Harriot and the Structure of Equations	248
	Pierre de Fermat and the <i>Introduction to Plane and Solid Loci</i>	253
	Albert Girard and the Fundamental Theorem of Algebra	258

René Descartes and <i>The Geometry</i>	261
Johann Hudde and Jan de Witt, Two Commentators on <i>The Geometry</i>	271
Isaac Newton and the <i>Arithmetica universalis</i>	275
Colin Maclaurin's <i>Treatise of Algebra</i>	280
Leonhard Euler and the Fundamental Theorem of Algebra	283

Finding the Roots of Algebraic Equations	289
The Eighteenth-Century Quest to Solve Higher-Order Equations Algebraically	290
The Theory of Permutations	300
Determining Solvable Equations	303
The Work of Galois and Its Reception	310
The Many Roots of Group Theory	317
The Abstract Notion of a Group	328

Understanding Polynomial Equations in n Unknowns	335
Solving Systems of Linear Equations in n Unknowns	336
Linearly Transforming Homogeneous Polynomials in n Unknowns: Three Contexts	345
The Evolution of a Theory of Matrices and Linear Transformations	356
The Evolution of a Theory of Invariants	366

Understanding the Properties of "Numbers"	381
New Kinds of "Complex" Numbers	382
New Arithmetics for New "Complex" Numbers	388
What Is Algebra?: The British Debate	399
An "Algebra" of Vectors	408
A Theory of Algebras, Plural	415

The Emergence of Modern Algebra	427
Realizing New Algebraic Structures Axiomatically	430
The Structural Approach to Algebra	438

References	449
------------	-----

Index	477
-------	-----