

FROM THE PREFACE

Chapters 1 through 6 of this book cover the standard topics in multivariate calculus and a first course in linear algebra. The book can also be used for a course in analysis, using the proofs in the appendix.

The organization and selection of material differs from the standard approach in three ways, reflecting the following guiding principles.

First, we believe that at this level linear algebra should be more a convenient setting and language for multivariate calculus than a subject in its own right. The guiding principle of this unified approach is that, locally, a nonlinear function behaves like its derivative.

Second, we emphasize computationally effective algorithms, and we prove theorems by showing that these algorithms work.

Third, we use differential forms to generalize the fundamental theorem of calculus to higher dimensions. The great conceptual simplification gained by doing electromagnetism in the language of forms is a central motivation for using forms.

PRAISE FOR PREVIOUS EDITIONS

"I was very impressed with the depth, clarity and ambition of this book. It respects its readers, it assumes that they are intelligent and naturally curious about beautiful mathematics. Then it provides them with all the tools necessary to learn multivariable calculus, linear algebra and basic analysis." —review of third edition, MAA Reviews

"...a real gem. It has a breadth and depth that is rarely seen in undergraduate texts, and it teaches real mathematics from a researcher's point of view instead of the standard off-the-shelf recipes that have little use outside the classroom" —review of the second edition in the MAA Monthly, October 2003

"A must-have that will become a classic." —Professor Ricardo Perez-Marco, UCLA Department of Mathematics.

"A great book. I love books that cut through the smog and show that math is not all that hard. It is a bonus when they are entertaining." —Dr. Ralph Kelsey, Ohio University

"...if these topics were taught to physicists out of your book rather than through the standard physics curriculum, much time and aggravation could be saved." —Brian Beckman, Microsoft

"The book's chief asset is its overall structure and philosophy; it does things right. It is the unique tactic of engaging rather than insulting the students' intelligence that makes the book great." —Professor Robert Ghrist, University of Pennsylvania

"A gold mine of information not available in my other texts." —Professor Thomas Tredon, Lord Fairfax Community College



ISBN 978-0-9715766-8-1



Contents

PREFACE	vii
CHAPTER 0 PRELIMINARIES	
0.0 Introduction	1
0.1 Reading mathematics	1
0.2 Quantifiers and negation	4
0.3 Set theory	6
0.4 Functions	9
0.5 Real numbers	17
0.6 Infinite sets	22
0.7 Complex numbers	25
CHAPTER 1 VECTORS, MATRICES, AND DERIVATIVES	
1.0 Introduction	32
1.1 Introducing the actors: Points and vectors	33
1.2 Introducing the actors: Matrices	42
1.3 Matrix multiplication as a linear transformation	56
1.4 The geometry of \mathbb{R}^n	67
1.5 Limits and continuity	83
1.6 Five big theorems	104
1.7 Derivatives in several variables as linear transformations	119
1.8 Rules for computing derivatives	137
1.9 The mean value theorem and criteria for differentiability	145
1.10 Review exercises for Chapter 1	152
CHAPTER 2 SOLVING EQUATIONS	
2.0 Introduction	159
2.1 The main algorithm: Row reduction	160
2.2 Solving equations with row reduction	166
2.3 Matrix inverses and elementary matrices	175
2.4 Linear combinations, span, and linear independence	180
2.5 Kernels, images, and the dimension formula	192
2.6 Abstract vector spaces	207
2.7 Eigenvectors and eigenvalues	219
2.8 Newton's method	232
2.9 Superconvergence	252
2.10 The inverse and implicit function theorems	258
2.11 Review exercises for Chapter 2	277

CHAPTER 3 MANIFOLDS, TAYLOR POLYNOMIALS, QUADRATIC FORMS, AND CURVATURE

3.0	Introduction	283
3.1	Manifolds	284
3.2	Tangent spaces	305
3.3	Taylor polynomials in several variables	314
3.4	Rules for computing Taylor polynomials	325
3.5	Quadratic forms	332
3.6	Classifying critical points of functions	342
3.7	Constrained critical points and Lagrange multipliers	349
3.8	Probability and the singular value decomposition	367
3.9	Geometry of curves and surfaces	378
3.10	Review exercises for Chapter 3	396

CHAPTER 4 INTEGRATION

4.0	Introduction	401
4.1	Defining the integral	402
4.2	Probability and centers of gravity	417
4.3	What functions can be integrated?	424
4.4	Measure zero	430
4.5	Fubini's theorem and iterated integrals	438
4.6	Numerical methods of integration	449
4.7	Other pavings	459
4.8	Determinants	461
4.9	Volumes and determinants	479
4.10	The change of variables formula	486
4.11	Lebesgue integrals	498
4.12	Review exercises for Chapter 4	520

CHAPTER 5 VOLUMES OF MANIFOLDS

5.0	Introduction	524
5.1	Parallelograms and their volumes	525
5.2	Parametrizations	528
5.3	Computing volumes of manifolds	538
5.4	Integration and curvature	550
5.5	Fractals and fractional dimension	560
5.6	Review exercises for Chapter 5	562

CHAPTER 6 FORMS AND VECTOR CALCULUS

6.0	Introduction	564
6.1	Forms on \mathbb{R}^n	565
6.2	Integrating form fields over parametrized domains	577
6.3	Orientation of manifolds	582

6.4	Integrating forms over oriented manifolds	589
6.5	Forms in the language of vector calculus	599
6.6	Boundary orientation	611
6.7	The exterior derivative	626
6.8	Grad, curl, div, and all that	633
6.9	The pullback	640
6.10	The generalized Stokes's theorem	645
6.11	The integral theorems of vector calculus	661
6.12	Electromagnetism	669
6.13	Potentials	688
6.14	Review exercises for Chapter 6	699
APPENDIX: ANALYSIS		
A.0	Introduction	704
A.1	Arithmetic of real numbers	704
A.2	Cubic and quartic equations	708
A.3	Two results in topology: Nested compact sets and Heine-Borel	713
A.4	Proof of the chain rule	715
A.5	Proof of Kantorovich's theorem	717
A.6	Proof of Lemma 2.9.5 (superconvergence)	723
A.7	Proof of differentiability of the inverse function	724
A.8	Proof of the implicit function theorem	729
A.9	Proving the equality of crossed partials	732
A.10	Functions with many vanishing partial derivatives	733
A.11	Proving rules for Taylor polynomials; big O and little o	735
A.12	Taylor's theorem with remainder	740
A.13	Proving Theorem 3.5.3 (completing squares)	745
A.14	Classifying constrained critical points	746
A.15	Geometry of curves and surfaces: Proofs	750
A.16	Stirling's formula and proof of the central limit theorem	756
A.17	Proving Fubini's theorem	760
A.18	Justifying the use of other pavings	762
A.19	Change of variables formula: A rigorous proof	765
A.20	Volume 0 and related results	772
A.21	Lebesgue measure and proofs for Lebesgue integrals	776
A.22	Computing the exterior derivative	794
A.23	Proving Stokes's theorem	797
BIBLIOGRAPHY		804
PHOTO CREDITS		805
INDEX		807