

# INDRA'S PEARLS

Felix Klein, one of the great nineteenth-century geometers, discovered in mathematics an idea prefigured in Buddhist mythology: the heaven of Indra contained a net of pearls, each of which was reflected in its neighbour, so that the whole Universe was mirrored in each pearl. Klein studied infinitely repeated reflections and was led to forms with multiple co-existing symmetries, each simple in itself, but whose interactions produce fractals on the edge of chaos. For a century these images, which were practically impossible to draw by hand, barely existed outside the imagination of mathematicians. However in the 1980s the authors embarked on the first computer exploration of Klein's vision, and in so doing found further extraordinary images of their own. Join the authors on the path from some basic mathematical ideas to the simple algorithms that create the delicate fractal filigrees, most of which have never appeared in print before. Beginners can learn to understand what the images mean and follow the step-by-step instructions for writing computer programs that generate them. More advanced readers can see how the images relate to ideas that take them to the forefront of research.

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## CONTENTS

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<i>Preface</i>	vii
<i>Introduction</i>	xv
1 The language of symmetry	1
2 A delightful fiction	36
3 Double spirals and Möbius maps	62
4 The Schottky dance	96
5 Fractal dust and infinite words	121
6 Indra's necklace	157
7 The glowing gasket	196
8 Playing with parameters	224
9 Accidents will happen	268
10 Between the cracks	310
11 Crossing boundaries	353
12 Epilogue	373
<i>Index</i>	393
<i>Road map</i>	396