

About Teaching Mathematics is the most important resource in my teaching library.

—Rusty Bresser, Lecturer and Supervisor of Teacher Education,
Education Studies Department, University of California, San Diego

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—Cathy Seeley, NCTM Past President and author,
Faster Isn't Smarter and Smarter Than We Think

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Part 1, "Starting Points," reflects the major overhaul of this book and addresses twenty-three issues that are important to thinking about teaching mathematics today.

Part 2, "Problem-Solving Investigations," opens with how to plan problem-solving lessons, followed by whole-class, small-group, and individual investigations organized into five areas of the curriculum: Measurement, Data, Geometry, Patterns and Algebraic Thinking, and Number and Operations.

Part 3, "Teaching Arithmetic," focuses on the cornerstone of elementary mathematics curriculum, offering ideas and assessments that build students' understanding, confidence, and competence in arithmetic.

Part 4, "Questions Teachers Ask," features Marilyn's responses to pedagogical questions she's received from teachers over the years.

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Marilyn Burns is one of today's most highly respected mathematics educators. Over the course of more than fifty years, Marilyn has taught children, led in-service sessions, spoken at conferences, contributed to professional journals, written more than a dozen books for children, and created more than twenty professional development resources for teachers and administrators. She is the founder of Math Solutions and a recipient of numerous awards including an honorary doctorate degree from Bank Street College of Education in New York, the Glenn Gilbert National Leadership Award from the National Council of Supervisors of Mathematics, and the Louise Hay Award for Contributions to Mathematics Education from the Association for Women in Mathematics.

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This icon indicates that I've written a mathematical commentary for the investigation. I've written these explanations in response to queries from teachers asking for math help with particular problems. I encourage you to try the investigation first before reading my explanation.

—MKB

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