

This is a book on symplectic topology, a rapidly developing field of mathematics which originated as a geometric tool for problems of classical mechanics. Since the 1980s, powerful methods such as Gromov's pseudo-holomorphic curves and Morse-Floer theory on loop spaces gave rise to the discovery of unexpected symplectic phenomena. The present book focuses on function spaces associated with a symplectic manifold. A number of recent advances show that these spaces exhibit intriguing properties and structures, giving rise to an alternative intuition and new tools in symplectic topology. The book provides an essentially self-contained introduction into these developments along with applications to symplectic topology, algebra and geometry of symplectomorphism groups, Hamiltonian dynamics and quantum mechanics. It will appeal to researchers and students from the graduate level onwards.

I like the spirit of this book. It formulates concepts clearly and explains the relationship between them. The subject matter is important and interesting.

—**Dusa McDuff, Barnard College, Columbia University**

This is a very important book, coming at the right moment. The book is a remarkable mix of introductory chapters and research topics at the very forefront of actual research. It is full of cross fertilizations of different theories, and will be useful to Ph.D. students and researchers in symplectic geometry as well as to many researchers in other fields (geometric group theory, functional analysis, mathematical quantum mechanics). It is also perfectly suited for a Ph.D.-students seminar.

—**Felix Schlenk, Université de Neuchâtel**

ISBN 978-1-4704-1693-5



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