

The International Mathematical Olympiad (IMO) is an annual international mathematics competition held for pre-collegiate students. It is also the oldest of the international science olympiads, and competition for places is particularly fierce. This book is an amalgamation of the booklets originally produced to guide students intending to contend for placement on their country's IMO team. See also *A First Step to Mathematical Olympiad Problems* which was published in 2009.

The material contained in this book provides an introduction to the main mathematical topics covered in the IMO, which are: Combinatorics, Geometry and Number Theory. In addition, there is a special emphasis on how to approach unseen questions in Mathematics, and model the writing of proofs. Full answers are given to all questions.

Though *A Second Step to Mathematical Olympiad Problems* is written from the perspective of a mathematician, it is written in a way that makes it easily comprehensible to adolescents. This book is also a must-read for coaches and instructors of mathematical competitions.

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A Second Step to Mathematical Olympiad Problems



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