**Statistical Inference: An Integrated Approach, Second Edition** presents an account of the Bayesian and frequentist approaches to statistical inference. Now with an additional author, this second edition places a more balanced emphasis on both perspectives than the first edition.

**New to the Second Edition**

- New material on empirical Bayes and penalized likelihoods and their impact on regression models
- Expanded material on hypothesis testing, method of moments, bias correction, and hierarchical models
- More examples and exercises
- More comparison between the approaches, including their similarities and differences

The text thoroughly covers statistical inference without delving too deep into technical details. It compares the Bayesian and frequentist schools of thought and explores procedures that lie on the border between the two. Many examples illustrate the methods and models, and exercises are included at the end of each chapter.

**Features**

- Provides readers with an integrated understanding of statistical inference from both classical and Bayesian perspectives
- Describes the strengths and weaknesses of the two viewpoints, emphasizing the importance of a comparative approach to inference
- Covers all the main topics of inference, including point and interval estimation, hypothesis testing, prediction, approximation, and linear models
- Includes real data examples and numerous exercises

K13686

CRC Press
Taylor & Francis Group
an **informa** business
www.crcpress.com

6000 Broken Sound Parkway, NW
Suite 300, Boca Raton, FL 33487
711 Third Avenue
New York, NY 10017
2 Park Square, Milton Park
Abingdon, Oxon OX14 4RN, UK

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