SCIENCE/PHYSICS

GEOMETRY AND LIGHT

The Science of Invisibility

Ulf Leonhardt and Thomas Philbin

"A startlingly original, historically erudite and elegantly written reinterpretation of classical ray and wave optics in terms of geometrical transformations of space, yielding new insights into familiar phenomena and designs for perfect imaging and concealment devices."—Sir Michael Berry, University of Bristol, United Kingdom

The science of invisibility combines two of physics' greatest concepts: Einstein's general relativity and Maxwell's principles of electromagnetism. Recent years have witnessed major breakthroughs in the area, and the authors of this volume—Ulf Leonhardt and Thomas Philbin of Scotland's University of St. Andrews—have been active in the transformation of invisibility from fiction into science. Their work on designing invisibility devices is based on modern metamaterials, inspired by Fermat's principle, analogies between mechanics and optics, and the geometry of curved space.

Suitable for graduate students and advanced undergraduates of engineering, physics, or mathematics, and scientific researchers of all types, this is the first authoritative textbook on invisibility and the science behind it. The book is two books in one: it introduces the mathematical foundations—differential geometry—for physicists and engineers, and it shows how concepts from general relativity become practically useful in electrical and optical engineering, not only for invisibility but also for perfect imaging and other fascinating topics. More than one hundred full-color illustrations and exercises with solutions complement the text.

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Contents

CHAPTER 1 Prologue 1

CHAPTER 2 Fermat's principle 7

1. Letters from Pierre de Fermat 7
2. Variational calculus 15
3. Newtonian analogy 20
4. Hamilton's equations 27
5. Optical conformal mapping 33
6. Transmutation 44
7. Spherical symmetry 48
8. Tomography 58
9. From invisible spheres to perfect lenses 66

CHAPTER 3 Differential geometry 83

10. Coordinate transformations 83
11. The metric tensor 87
12. Vectors and bases 92
13. One–forms and general tensors 99
14. Vector products and the Levi–Civita tensor 103
15. The covariant derivative of a vector 107
16. Covariant derivatives of tensors and of the metric 115
17. Divergence, curl and Laplacian 119
18. Curvature 126
19. Geodesics 129
20. Parallel transport and covariant derivatives 133
21. Geodesic deviation and the Riemann tensor 136
22. Parallel transport around a closed loop 144
23. Conformally flat spaces 151
24. The hypersphere 155
25. Space–time geometry 161

CHAPTER 4 Maxwell's equations 165

26. Spatial geometries and media 166
27. Planar media 170
28. Transformation media 173
29. Electromagnetic waves 175
30. Geometrical optics 186
31. Space–time geometries and media 199

CHAPTER 5 Geometries and media 205

32. Spatial transformation media 206
33. Perfect invisibility devices 210
34. Negative refraction and perfect lenses 216
35. Cloaking at a distance 221
36. Perfect imaging with positive refraction 227
37. Moving media 239
38. Optical Aharonov–Bohm effect 246
39. Analogue of the event horizon 251

Appendix 261

Bibliography 265