

Table of Contents

Preface	vii
0 Introduction	1
0.1 Free semigroups, rational languages	1
0.2 Recognizability by finite automata and by finite semigroups	3
0.3 Syntactical classification of rational languages? . . .	5
0.4 Typical questions involving pseudovarieties	6
I Finite Universal Algebra	9
1 Elements of Universal Algebra	11
1.1 Algebraic types	11
1.2 Homomorphisms, congruences, subalgebras, direct products	19
1.3 Free algebras, varieties, identities	24
1.4 Completeness of equational logic	28
2 Order and Topology	33
2.1 Well quasi-orderings	33
2.2 Uniform structures	45
2.3 The completion of partially ordered sets	48
3 Finite Algebras	53
3.1 Pseudovarieties; recognizability of subsets of free algebras	53
3.2 Definition of pseudovarieties by filters of identities .	60

3.3	Eilenberg-type correspondences	65
3.4	Implicit operations and their topological algebra	71
3.5	Definition of pseudovarieties by pseudoidentities	81
3.6	Topological characterization of recognizability	85
3.7	Examples	87
3.8	Closed sets of pseudoidentities	96
4	Decidability	105
4.1	Algorithms	105
4.2	Word problems	107
4.3	Finite bases and decidability	112
II	Finite Semigroups and Monoids	121
5	Preliminaries	123
5.1	Green's relations	123
5.2	Some important examples of pseudovarieties	129
5.3	Semigroups of transformations	133
5.4	Combinatorics	135
5.5	Bands	136
5.6	Pointwise properties of implicit operations	140
5.7	Graphs	142
6	Permutativity	149
6.1	Varieties of commutative semigroups	149
6.2	Pseudovarieties of commutative semigroups	160
6.3	Linear identities	167
6.4	Permutative semigroups	175
6.5	Minimal non permutative pseudovarieties	181
7	Operators Relating Semigroups and Monoids	201
7.1	Four natural operators	201
7.2	Some calculations of MV	208
8	Semigroups Whose Regular \mathcal{D}-Classes are Subsemigroups	215
8.1	Implicit operations on DS	215

8.2	Implicit operations on J	224
8.3	Some extensions of J	234
9	The Join	241
9.1	The join G \vee Com	242
9.2	The join R \vee L	246
9.3	Join decompositions	254
10	The Semidirect Product	265
10.1	Semidirect product and wreath product	265
10.2	Representation of free objects	272
10.3	Iterated semidirect product of semilattices	278
10.4	Locally trivial pseudovarieties closed under semidirect product	288
10.5	The Krohn-Rhodes decomposition	294
10.6	Semidirect products of the form V * D _k	299
10.7	The pseudovariety Com * D	311
10.8	The pseudovarieties Com _{1,k} * D _n	322
10.9	The pseudovariety J * D	328
10.10	Pseudovarieties closed under semidirect product	341
11	The Power	357
11.1	Two examples	358
11.2	The operators P and P' and their counterparts for varieties of languages	360
11.3	Identities satisfied by power semigroups	363
11.4	Locally trivial pseudovarieties	368
11.5	Permutative pseudovarieties	371
11.6	Non permutative pseudovarieties	375
11.7	\mathcal{R} -trivial power pseudovarieties	383
11.8	The pseudovariety PJ	395
11.9	Locally commutative power pseudovarieties	404
11.10	Pseudovarieties of aperiodic monoids	412
11.11	Completely regular pseudovarieties	425

12 Factorization of Implicit Operations	429
12.1 Irreducible implicit operations	429
12.2 Chain conditions for principal ideals	433
12.3 An "exotic" example: $\bar{\Omega}_n LSI$	436
Open Problems	441
Bibliographic Notes	447
Bibliography	459
Author index	495
Index	501
Notation	507