

A Comprehensive Course in Analysis by Poincaré Prize winner Barry Simon is a five-volume set that can serve as a graduate-level analysis textbook with a lot of additional *bonus* information, including hundreds of problems and numerous notes that extend the text and provide important historical background. Depth and breadth of exposition make this set a valuable reference source for almost all areas of classical analysis.



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Part 3 returns to the themes of Part 1 by discussing pointwise limits (going beyond the usual focus on the Hardy-Littlewood maximal function by including ergodic theorems and martingale convergence), harmonic functions and potential theory, frames and wavelets, H^p spaces (including bounded mean oscillation (BMO)) and, in the final chapter, lots of inequalities, including Sobolev spaces, Calderon-Zygmund estimates, and hypercontractive semigroups.

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