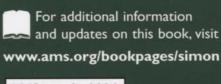
A Comprehensive Course in Analysis by Poincaré Prize winner Barry Simon is a five-volume set that can serve as a graduate-level analysis textbook with a lot of additional bonus information, including hundreds of problems and numerous notes that extend the text and provide important historical background. Depth and breadth of exposition make this set a valuable reference source for almost all areas of classical analysis.



Photo courtesy of Bob Paz, Caltech

Part 3 returns to the themes of Part 1 by discussing pointwise limits (going beyond the usual focus on the Hardy-Littlewood maximal function by including ergodic theorems and martingale convergence), harmonic functions and potential theory, frames and wavelets, H^p spaces (including bounded mean oscillation (BMO)) and, in the final chapter, lots of inequalities, including Sobolev spaces, Calderon-Zygmund estimates, and hypercontractive semigroups.





AMS on the Web www.ams.org

Contents

Preface to	the Series	XI
Preface to	Part 3	xvii
Chapter 1.	Preliminaries	1
§1.1.	Notation and Terminology	1
§1.2.	Some Results for Real Analysis	3
§1.3.	Some Results from Complex Analysis	12
§1.4.	Green's Theorem	16
Chapter 2.	Pointwise Convergence Almost Everywhere	19
§2.1.	The Magic of Maximal Functions	22
§2.2.	Distribution Functions, Weak- L^1 , and Interpolation	26
§2.3.	The Hardy-Littlewood Maximal Inequality	41
§2.4.	Differentiation and Convolution	52
§2.5.	Comparison of Measures	60
§2.6.	The Maximal and Birkhoff Ergodic Theorems	65
§2.7.	Applications of the Ergodic Theorems	92
§2.8.	Bonus Section: More Applications of the Ergodic Theorems	102
§2.9.	Bonus Section: Subadditive Ergodic Theorem and Lyapunov Behavior	133
§2.10.	Martingale Inequalities and Convergence	147
§2.11.	The Christ–Kiselev Maximal Inequality and Pointwise Convergence of Fourier Transforms	168

vii

Chapter 3.	Harmonic and Subharmonic Functions	173
§3.1.	Harmonic Functions	177
§3.2.	Subharmonic Functions	202
§3.3.	Bonus Section: The Eremenko–Sodin Proof of Picard's Theorem	213
§3.4.	Perron's Method, Barriers, and Solution of the Dirichlet Problem	220
§3.5.	Spherical Harmonics	232
§3.6.	Potential Theory	252
§3.7.	Bonus Section: Polynomials and Potential Theory	278
§3.8.	Harmonic Function Theory of Riemann Surfaces	298
Chapter 4.	Bonus Chapter: Phase Space Analysis	319
§4.1.	The Uncertainty Principle	320
§4.2.	The Wavefront Sets and Products of Distributions	345
§4.3.	Microlocal Analysis: A First Glimpse	352
§4.4.	Coherent States	373
§4.5.	Gabor Lattices	390
§4.6.	Wavelets	407
Chapter 5.	${\cal H}^p$ Spaces and Boundary Values of Analytic Functions on the Unit Disk	437
§5.1.	Basic Properties of H^p	439
§5.2.	H^2 belongered that / State / State / Senother with a substitution H^2	444
§5.3.	First Factorization (Riesz) and H^p	450
§5.4.	Carathéodory Functions, h^1 , and the Herglotz	
	Representation	459
§5.5.	Boundary Value Measures	464
§5.6.	Second Factorization (Inner and Outer Functions)	468
§5.7.	Conjugate Functions and M. Riesz's Theorem	472
§5.8.	Homogeneous Spaces and Convergence of Fourier Series	493
§5.9.	Boundary Values of Analytic Functions in the Upper Half-Plane	498
§5.10.	Beurling's Theorem	515
§5.11.	H^p -Duality and BMO	517
§5.12.	Cotlar's Theorem on Ergodic Hilbert Transforms	539

Contents

Chapter 6	Bonus Chapter: More Inequalities	543
§6.1.	Lorentz Spaces and Real Interpolation	547
§6.2.	Hardy-Littlewood–Sobolev and Stein–Weiss Inequalities	559
§6.3.	Sobolev Spaces; Sobolev and Rellich–Kondrachov Embedding Theorems	565
§6.4.	The Calderón–Zygmund Method	588
§6.5.	Pseudodifferential Operators on Sobolev Spaces and the Calderón–Vaillancourt Theorem	604
§6.6.	Hypercontractivity and Logarithmic Sobolev Inequalities	615
§6.7.	Lieb-Thirring and Cwikel-Lieb-Rosenblum Inequalities	657
§6.8.	Restriction to Submanifolds	671
§6.9.	Tauberian Theorems	686
Bibliography		691
Symbol Index		737
Subject Index		739
Author Index		751
Index of Capsule Biographies		759