A Comprehensive Course in Analysis by Poincaré Prize winner Barry Simon is a five-volume set that can serve as a graduate-level analysis textbook with a lot of additional *bonus* information, including hundreds of problems and numerous notes that extend the text and provide important historical background. Depth and breadth of exposition make this set a valuable reference source for almost all areas of classical analysis.

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Part 2B provides a comprehensive look at a number of subjects of complex analysis not included in Part 2A. Presented in this volume are the theory of conformal metrics (including the Poincaré metric, the Ahlfors-Robinson proof of Picard's theorem, and Bell's proof of the Painlevé smoothness theorem), topics in analytic number theory (including Jacobi's two- and four-square theorems, the Dirichlet prime progression theorem, the prime number theorem, and the Hardy-Littlewood asymptotics for the number of partitions), the theory of Fuschian differential equations, asymptotic methods (including Euler's method, stationary phase, the saddle-point method, and the WKB method), univalent functions (including an introduction to SLE), and Nevanlinna theory. The chapters on Fuschian differential equations and on asymptotic methods can be viewed as a minicourse on the theory of special functions.

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