

A Comprehensive Course in Analysis by Poincaré Prize winner Barry Simon is a five-volume set that can serve as a graduate-level analysis textbook with a lot of additional *bonus* information, including hundreds of problems and numerous notes that extend the text and provide important historical background. Depth and breadth of exposition make this set a valuable reference source for almost all areas of classical analysis.



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Part 1 is devoted to real analysis. From one point of view, it presents the infinitesimal calculus of the twentieth century with the ultimate integral calculus (measure theory) and the ultimate differential calculus (distribution theory). From another, it shows the triumph of abstract spaces: topological spaces, Banach and Hilbert spaces, measure spaces, Riesz spaces, Polish spaces, locally convex spaces, Fréchet spaces, Schwartz space, and L^p spaces. Finally it is the study of big techniques, including the Fourier series and transform, dual spaces, the Baire category, fixed point theorems, probability ideas, and Hausdorff dimension. Applications include the constructions of nowhere differentiable functions, Brownian motion, space-filling curves, solutions of the moment problem, Haar measure, and equilibrium measures in potential theory.

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