

TEXTBOOKS in MATHEMATICS

"Krantz is a very prolific writer ...[who] creates excellent examples and problem sets."

—Albert Boggess, Professor and Director of the School of Mathematics and Statistical Sciences, Arizona State University, Tempe, USA

Designed for a one- or two-semester undergraduate course, **Differential Equations: Theory, Technique, and Practice, Second Edition** educates a new generation of mathematical scientists and engineers on differential equations. This edition continues to emphasize examples and mathematical modeling as well as promote analytical thinking to help students in future studies.

New to the Second Edition

- Improved exercise sets and examples
- Reorganized material on numerical techniques
- Enriched presentation of predator-prey problems
- Updated material on nonlinear differential equations and dynamical systems
- A new appendix that reviews linear algebra

In each chapter, lively historical notes and mathematical nuggets enhance students' reading experience by offering perspectives on the lives of significant contributors to the discipline. "Anatomy of an Application" sections highlight rich applications from engineering, physics, and applied science. Problems for review and discovery also give students some open-ended material for exploration and further learning.

About the Author

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Table of Contents

Preface to the Second Edition	xiii
Preface to the First Edition	xv
1 What Is a Differential Equation?	1
1.1 Introductory Remarks	1
1.2 A Taste of Ordinary Differential Equations	4
1.3 The Nature of Solutions	6
1.4 Separable Equations	13
1.5 First-Order Linear Equations	16
1.6 Exact Equations	20
1.7 Orthogonal Trajectories and Curves	25
1.8 Homogeneous Equations	30
1.9 Integrating Factors	34
1.10 Reduction of Order	37
1.10.1 Dependent Variable Missing	38
1.10.2 Independent Variable Missing	39
1.11 The Hanging Chain and Pursuit Curves	42
1.11.1 The Hanging Chain	42
1.11.2 Pursuit Curves	47
1.12 Electrical Circuits	51
Anatomy of an Application	55
Problems for Review and Discovery	59
2 Second-Order Linear Equations	63
2.1 Second-Order Linear Equations with Constant Coefficients	63
2.2 The Method of Undetermined Coefficients	69
2.3 The Method of Variation of Parameters	74
2.4 The Use of a Known Solution to Find Another	78
2.5 Vibrations and Oscillations	81
2.5.1 Undamped Simple Harmonic Motion	82
2.5.2 Damped Vibrations	84
2.5.3 Forced Vibrations	87
2.5.4 A Few Remarks About Electricity	89

2.6	Newton's Law of Gravitation and Kepler's Laws	92
2.6.1	Kepler's Second Law	95
2.6.2	Kepler's First Law	97
2.6.3	Kepler's Third Law	100
2.7	Higher-Order Equations	105
	Historical Note	111
	Anatomy of an Application	113
	Problems for Review and Discovery	117
3	Qualitative Properties and Theoretical Aspects	119
3.1	A Bit of Theory	119
3.2	Picard's Existence and Uniqueness Theorem	126
3.2.1	The Form of a Differential Equation	126
3.2.2	Picard's Iteration Technique	127
3.2.3	Some Illustrative Examples	128
3.2.4	Estimation of the Picard Iterates	130
3.3	Oscillations and the Sturm Separation Theorem	131
3.4	The Sturm Comparison Theorem	143
	Anatomy of an Application	147
	Problems for Review and Discovery	152
4	Power Series Solutions and Special Functions	155
4.1	Introduction and Review of Power Series	155
4.1.1	Review of Power Series	156
4.2	Series Solutions of First-Order Equations	165
4.3	Ordinary Points	170
4.4	Regular Singular Points	178
4.5	More on Regular Singular Points	184
4.6	Gauss's Hypergeometric Equation	192
	Historical Note	197
	Historical Note	199
	Anatomy of an Application	201
	Problems for Review and Discovery	204
5	Numerical Methods	207
5.1	Introductory Remarks	208
5.2	The Method of Euler	208
5.3	The Error Term	212
5.4	An Improved Euler Method	216
5.5	The Runge-Kutta Method	220
	Anatomy of an Application	225
	Problems for Review and Discovery	228

6	Fourier Series: Basic Concepts	231
6.1	Fourier Coefficients	231
6.2	Some Remarks about Convergence	240
6.3	Even and Odd Functions: Cosine and Sine Series	246
6.4	Fourier Series on Arbitrary Intervals	251
6.5	Orthogonal Functions	255
	Historical Note	260
	Anatomy of an Application	262
	Problems for Review and Discovery	272
7	Partial Differential Equations and Boundary Value Problems	275
7.1	Introduction and Historical Remarks	275
7.2	Eigenvalues and the Vibrating String	279
7.2.1	Boundary Value Problems	279
7.2.2	Derivation of the Wave Equation	280
7.2.3	Solution of the Wave Equation	282
7.3	The Heat Equation	288
7.4	The Dirichlet Problem for a Disc	294
7.4.1	The Poisson Integral	297
7.5	Sturm—Liouville Problems	301
	Historical Note	306
	Historical Note	307
	Anatomy of an Application	309
	Problems for Review and Discovery	313
8	Laplace Transforms	317
8.0	Introduction	317
8.1	Applications to Differential Equations	320
8.2	Derivatives and Integrals	325
8.3	Convolutions	330
8.3.1	Abel's Mechanics Problem	333
8.4	The Unit Step and Impulse Functions	338
	Historical Note	346
	Anatomy of an Application	347
	Problems for Review and Discovery	350
9	The Calculus of Variations	355
9.1	Introductory Remarks	355
9.2	Euler's Equation	360
9.3	Isoperimetric Problems and the Like	369
9.3.1	Lagrange Multipliers	370
9.3.2	Integral Side Conditions	371
9.3.3	Finite Side Conditions	375
	Historical Note	379
	Anatomy of an Application	382
	Problems for Review and Discovery	386

10 Systems of First-Order Equations	389
10.1 Introductory Remarks	389
10.2 Linear Systems	392
10.3 Systems with Constant Coefficients	400
10.4 Nonlinear Systems	407
Anatomy of an Application	414
Problems for Review and Discovery	419
11 The Nonlinear Theory	423
11.1 Some Motivating Examples	424
11.2 Specializing Down	424
11.3 Types of Critical Points: Stability	428
11.4 Critical Points and Stability	438
11.5 Stability by Liapunov's Direct Method	448
11.6 Simple Critical Points of Nonlinear Systems	454
11.7 Nonlinear Mechanics: Conservative Systems	460
11.8 Periodic Solutions	466
Historical Note	476
Anatomy of an Application	478
Problems for Review and Discovery	481
12 Dynamical Systems	485
12.1 Flows	486
12.1.1 Dynamical Systems	488
12.1.2 Stable and Unstable Fixed Points	490
12.1.3 Linear Dynamics in the Plane	491
12.2 Some Ideas from Topology	499
12.2.1 Open and Closed Sets	499
12.2.2 The Idea of Connectedness	500
12.2.3 Closed Curves in the Plane	502
12.3 Planar Autonomous Systems	504
12.3.1 Ingredients of the Proof of Poincaré–Bendixson	505
Anatomy of an Application	516
Problems for Review and Discovery	521
Appendix: Review of Linear Algebra	523
Bibliography	533
Index	535