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$\mathcal{W}$ -SYMMETRY

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$\mathcal{W}$ -symmetry is an extension of conformal symmetry in two dimensions. Since its introduction in 1985, $\mathcal{W}$ -symmetry has become one of the central notions in the study of two-dimensional conformal field theory. The mathematical structures that underlie $\mathcal{W}$ -symmetry are so-called $\mathcal{W}$ -algebras, which are higher-spin extensions of the Virasoro algebra. This book contains a collection of papers on $\mathcal{W}$ -symmetry, covering the period from 1985 through 1993. Its main focus is the construction of $\mathcal{W}$ -algebras and their representation theory. A recurrent theme is the intimate connection between $\mathcal{W}$ -algebras and affine Lie algebras. Some of the applications, in particular $\mathcal{W}$ -gravity, are also covered.

About the editors

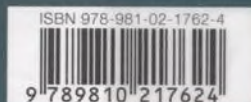
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