

String Theory in a Nutshell

This book is the essential new introduction to modern string theory, by one of the world's authorities on the subject. Concise, clearly presented, and up-to-date, *String Theory in a Nutshell* brings together the best understood and most important aspects of a theory that has been evolving since the early 1980s. A core model of physics that substitutes one-dimensional extended "strings" for zero-dimensional point-like particles (as in quantum field theory), string theory has been the leading candidate for a theory that would successfully unify all fundamental forces of nature, including gravity.

Starting with the basic definitions of the theory, Elias Kiritsis guides readers through classic and modern topics. In particular, he treats perturbative string theory and its Conformal Field Theory (CFT) tools in detail while also developing nonperturbative aspects and exploring the unity of string interactions. He presents recent topics including black holes, their microscopic entropy, and the AdS/CFT correspondence. He also describes matrix model tools for string theory. In all, the book contains nearly five hundred exercises for the graduate-level student, and works as a self-contained and detailed guide to the literature.

String Theory in a Nutshell is the staple one-volume reference on the subject not only for students and researchers of theoretical high-energy physics, but also for mathematicians and physicists specializing in theoretical cosmology and QCD.

"There is a definite need for a short speedy intro-

duction to modern string theory. Kiritsis beautifully fills this gap—including all essential areas, but remaining relatively concise, so that a beginning student can work through the entire text."

—Andrew Strominger, *Harvard University*

"An excellent reference for any graduate student interested in string theory. Kiritsis succinctly describes many of the recent developments that are necessary background to current research. Topics covered include black holes in string theory, holography, various dualities among string theories, and dualities connecting string theory to gauge theories. The basic frameworks for connecting string theory to four-dimensional physics are also explained."

—Juan Maldacena, *Institute for Advanced Study*

"This very well-written book, which builds on the fundamentals and provides an excellent introduction to the state of the art in string theory, will be quite useful to students and to researchers acquainting themselves with this exciting field. It concisely lays out the successes of string theory to date and the challenges that await. I have no doubt that the topics described herein will remain at the heart of the theory even when our understanding of its dynamics and its role in describing nature improve."

—David Kutasov, *University of Chicago*

Elias Kiritsis is Directeur de Recherche at the CNRS, affiliated with the École Polytechnique in Paris, and Professor of Physics at the University of Crete.

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