

Greiner  
Reinhardt  
**Quantum Electrodynamics**  
*Fourth Edition*

This textbook on *Quantum Electrodynamics* is a thorough introductory text providing all necessary mathematical tools together with many examples and worked problems. In their presentation of the subject the authors adopt a heuristic approach based on the propagator formalism. The latter is introduced in the first two chapters in both its nonrelativistic and relativistic versions. Subsequently, a large number of scattering and radiation processes involving electrons, positrons, and photons are introduced and their theoretical treatment is presented in great detail. Higher order processes and renormalization are also included. The book concludes with a discussion of two-particle states and the interaction of spinless bosons. This completely revised and corrected new edition provides several additions to enable deeper insight in formalism and application of quantum electrodynamics.

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