

Lecture Notes in Physics 769

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Editor

Physics of Black Holes

A Guided Tour

Black Holes are still considered to be among the most mysterious and fascinating objects in our universe. Awaiting the era of gravitational astronomy, much progress in theoretical modeling and understanding of classical and quantum black holes has been achieved.

The present volume serves as a tutorial, high-level guided tour through the black hole landscape: information paradox and blackhole thermodynamics, numerical simulations of black hole formation and collisions, braneworld scenarios and stability of black holes with respect to perturbations are treated in great detail, as is their possible occurrence at the LHC.

An outgrowth of a topical and tutorial summer school, this extensive set of carefully edited notes has been set up with the aim of constituting an advanced-level, multi-authored textbook which meets the needs of both postgraduate students and young researchers in the fields of modern cosmology, astrophysics and (quantum) field theory.

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