

Propositional logics usually do not incorporate the dimension of time. However, even Aristotle already mentioned that time plays an important role in the evaluation of truth values of propositions. After Aristotle's time, a lot was created by men and, nowadays, logic is not an exceptional area for human reasoning. Beginning in the 1940's, computers were built and the era of Artificial Intelligence gently started.

Nowadays, practically any more advanced product contains some kind of processor which takes decisions in a way similar to that of a human being. However, for such technical devices the forecast for truth values of propositions in the future is not only a speculation. Due to the constructions and the technical possibilities, we can often compute these values, and propositions concerning the near future are of great importance. This has motivated many authors to investigate the so-called temporal logic, i.e., the logic where time is considered as a variable of the propositional formula. The logical language of tense logic contains, in addition to the usual truth-functional operators, four so-called modal operators.

The aim of this monograph is to present an algebraic approach to an axiomatization of tense operators which are the most powerful tools in every tense logic. The authors hope that their monograph will be an incentive for further research.

Contents

1	Introduction	5
2	Preliminaries	11
2.1	Relations and ordered sets	11
2.2	Algebras	16
2.3	Semilattices and lattices	19
2.4	Ideals and filters in lattices and their applications	21
2.5	Galois connections	23
2.6	Residuated structures	25
2.7	Modules over ordered structures	26
2.8	MacNeille completion	29
2.9	Heyting algebras	31
2.10	Effect algebras	33
3	Tense logics	41
4	Quantifiers	47
5	Tense Boolean algebras	53
5.1	The tense algebra	53
5.2	The construction of tense operators	55
5.3	Modal and necessity Boolean algebras	58
6	Dynamic algebras	63
6.1	Dynamic pairs	63
6.2	The construction	65
6.3	Relations between bounded poset morphisms	72
6.4	Representation of Galois connections	74
6.5	Set representation of dynamic order algebras	78
6.6	Representation of tense algebras	80
6.7	Dynamic order algebras and their MacNeille completion	84
6.8	Parallel worlds and representations of tense order algebras	85

7	Tense De Morgan posets	90
7.1	Tense De Morgan posets and their construction	90
7.2	Representation and approximation	93
7.3	Set representation of tense De Morgan posets	98
7.4	Set representation of tense orthocomplemented posets	101
8	Tense operators in distributive lattices	109
8.1	Tense, modal and necessity distributive algebras	109
8.2	Tense distributive De Morgan algebras	116
8.3	Tense operators in intuitionistic logic	120
8.4	Finitely approximable Heyting algebras	129
9	Tense operators in quantum logic	132
9.1	Tense operators in effect algebras and their construction	133
9.2	Representation and approximation	139
9.3	Tense operators on q-effect algebras	144
9.4	Tense operators on spaces of numerical events	155
10	Tense operators in fuzzy logic	168
10.1	Fuzzy dynamic pairs and fuzzy tense algebras	169
10.2	Fuzzy binary relations and general fuzzy Galois connections . . .	178
	Index	188
	Bibliography	195