

# Boulevard of Broken Symmetries

Effective Field Theories of Condensed Matter

This textbook covers the main topics in contemporary condensed matter physics in a modern and unified way, using quantum field theory in the functional integral approach. The book highlights symmetry aspects in acknowledging that much of the collective behaviors of condensed matter at low temperatures emerge above a nontrivial ground state, which spontaneously breaks the symmetry.

The emphasis is on effective field theories which provide an efficient and powerful description that is valid at long wavelengths and low frequencies. In conjunction with this, the modern approach towards renormalization is taken, whereby a wave number cutoff is introduced to set a scale beyond which the microscopic model under consideration ceases to be valid.

The unique and innovative character of this presentation, free of historical constraints, allows for a compact and self-contained treatment of the main topics in contemporary condensed matter physics.

## Key Features

- Begins by studying a simple microscopic model that captures the essential physics, and then uses powerful calculation tools to arrive at the effective theory.
- Physical properties are extracted from the effective theories, and their emerging nature special to the long-wavelength, low-frequency corner is emphasized.
- Incorporates the effect of unknown physics above the cutoff by redefining the parameters of the original theory.

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