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Many-Body Problems and Quantum Field Theory

The book introduces the concepts and methods of the topics on a level suitable for graduate students and researchers. The formalism is developed in close conjunction with the description of a number of physical systems: cohesion and dielectric properties of the electron gas, superconductivity, superfluidity, nuclear matter and nucleon pairing, matter and radiation, interaction of fields by particle exchange and mass generation. Emphasis is placed on analogies between the various systems rather than on advanced or specialized aspects, with the purpose of illustrating common ideas within different domains of physics. Starting from a basic knowledge of quantum mechanics and classical electromagnetism, the exposition is self-contained and explicitly details all steps of the derivations.

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