

Different quantum gravity models have been devised to study the quantum aspects of gravitational interaction. The investigation of these models requires the computation of the effective action — hence the significance of this book.

Effective Action in Quantum Gravity is divided into three parts. The first part is pedagogical in nature and contains an introduction to the field theoretical models. The second part explains the quantum theory of the interacting fields in curved space including renormalization groups and the asymptotic properties of grand unification theories at high curvature. In the third part the authors discuss the problems of quantized gravitational field theory, in particular the quantum theory of higher-derivative gravity, the quantum Kaluza–Klein theories and the quantum theory of strings and membranes.

This book is intended for postgraduate students and researchers in high-energy physics and gravitational theory. Although a knowledge of quantum field theory and gravity is assumed, *Effective Action in Quantum Gravity* can be read without reference to other books or papers.

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